Chinese Calendar

An astronomical approach

Wenceslao Segura

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εWT
Ediciones

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此書謹獻給我的親家 - 桂峰和林紹

這本書既是我對中國曆法的謙遜探索，也是向您們及您們的文化致上的一份敬意。願這份跨越國界的研究，成為我們家庭情感與文化交融的見證。

This book is dedicated to my in-laws, Guifeng and Lin Shao.

This work is both an exploration of the
Chinese calendar and a tribute to your culture.
May this cross-border study serve as a testament
to the blending of our family's bonds and cultural heritage.

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1.- Introduction

The Chinese calendar is utilized in China and surrounding countries, though the Gregorian calendar is officially used for civil purposes. It has very ancient origins and took on its current form in 1645.

The Spring Festival, or Chinese New Year, is the most significant day on the Chinese calendar. All Chinese communities celebrate it, and it is gradually becoming an international holiday.

Traditionally, and until the Xinhai Revolution of 1911, the Chinese calendar was an important document issued by a specialized department directly dependent on the emperor.

The splendour the Chinese calendar has had and continues to have is not reflected in its bibliography. Given its close relationship with Chinese astronomy, numerous works on the calendar's history exist, but there is very little technical and astronomical information on the current calendar.

Some excellent works have recently been published, such as those by Helmer Aslaksen and Yuk Tung Liu, which have clarified the calendar's obscurities and added to the scarce bibliographical references from the end of the 20th century. However, there are no books specifically dedicated to the calendar's technical operations. For this reason, we publish this work, as we understand there is a gap in the bibliography on such an important subject as the astronomical aspects of the Chinese calendar.

We have divided this book into three parts. Chapters 2-7 discuss general matters related to calendars and fundamental elements of astronomical theory. Chapters 8-15 focus solely on the Chinese calendar. Lastly, appendices contain various information.

The reader will notice that there are numerous repetitions. We have done this intentionally to clarify the topics discussed and allow independent reading of some chapters. The reader will also notice the frequent use of tables and graphs., which facilitates the understanding of some complex issues.

One issue with the Chinese calendar is the abundance of Chinese words. We have tried to minimize their use to create a book suitable for an international audience. For the representation of Chinese words, we opted to write them in pinyin without tonal accents and avoided using Chinese characters altogether. This approach aims to facilitate reading for those outside of the Chinese context. Any Chi-

nese words that do appear are in italics.

An important issue in any scientific book is the notation; we have adopted the following criteria:

* Dates are represented in the order year/month/day. For example, 2033/07/15 is July 15, 2033. Months and days always have two digits. If only two numbers appear in the date notation, they refer to month and day.
* Time is represented in the order hours:minutes:seconds. For example, 12:34:03 represents 12 hours, 34 minutes, 3 seconds, making the minutes and seconds always represented by two digits.
* Angles are represented in the order degrees-minutes-seconds. For example, 123-09-40 for 123 degrees, 9 minutes, 40 seconds, and as in the previous cases, minutes and seconds always have two digits.
* If only two numbers appear in the time or angle notation, they are hours (or degrees) and minutes.
* Since there is no era in the Chinese calendar, we have adopted the Christian or Western era, which originates in the birth of Jesus Christ. Sometimes, we represent the years before the beginning of the era with astronomical notation, that is, using negative values. On other occasions, we cite the years with the abbreviations BCE (before the common era) and CE (common era). In some cases, we have found using the abbreviations AD (*Anno Domini*) and AC (*ante Christum*) more appropriate.
* For the astronomical magnitudes used in the text, we have taken the usual symbols.
* Decimal numbers are represented by a point and thousands by a comma, for example, 2,454,235.78. However, if the number is inside a formula, instead of a comma, there will be a blank space, for example, 2 454 235.78. Year numbers do not have separators.

It may seem strange to those not trained in astronomy that, in this book, we assume the Earth's centre is fixed while the Moon and the Sun move relative to it. From a kinematic perspective, whether we consider the Earth or the Sun as a reference point makes no difference. However, it is more appropriate to assume the Earth is fixed and the Sun is moving, as we do to describe astronomical phenomena observed from the Earth.

One last warning: We have developed numerous software programs to analyze the Chinese calendar. Our results may not coincide with those of other authors due to different astronomical algorithms and a different use of the civil time scale.

2.- The calendars

Calendars play a crucial role in the functioning of civilized society. They serve multiple purposes, primarily enabling the planning of future activities while preserving the memory of past events, which is vital for social cohesion.

Calendars are often of religious origin because religions need to establish their celebrations in an unchangeable way, guaranteeing their continuity over time. In addition, calendars have an economic function because they anticipate seasonal changes, allowing the programming of agricultural and economic activities and, in general, the rest of human activities closely related to the weather.

The history of calendars shows that they are very conservative institutions in the sense that the population refuses to change them. This social behaviour has made it difficult to improve the calendar, although it has also prevented frequent changes that might not have been successful; history shows that attempts at calendar reforms had to be abandoned shortly after being implemented when they were found inadequate.

The calendar reforms that have been successful and thus have endured have been carried out under a strong, centralized power, as was the case with Julius Caesar's reform in 46 BC or the reform sponsored by Pope Gregory XIII in 1582. It should be noted that the successful calendar changes were accomplished because they solved problems with the old calendar. However, history shows that sometimes the changes represented setbacks rather than advances.

Calendars are closely linked to astronomy, to the point that hemerology or the science of calendars, is an astronomical discipline. We can say that a calendar is a simplified astronomical theory, capable of tracking, approximately, the movements of the Sun and the Moon.

Calendar and astronomy

To measure time, we need a periodic phenomenon and a means of counting the periods that have passed. Astronomy gives us three useful periods for measuring the time of human activities: the day, or the period of rotation of the Earth around its axis; the synodic period, or the time between two consecutive new Moons; and the year, or the time it takes the Earth to go around the Sun.

The periods used in calendars are calculated very accurately, but they are variable, with a dispersion that sometimes makes them unsuitable for calendars. For this reason, most calendars use not the true values of astronomical magnitudes but the mean values. For example, this is the case of the Gregorian or Persian calendars and in general of all computational calendars (see below).

The measure of time

One challenge that hemerology faces is the necessity of using two different time scales. Astronomical phenomena are measured using the terrestrial time scale, a uniform scale with the atomic second as its unit (as discussed in Chapter 4). In contrast, calendars operate on universal time, a non-uniform time scale defined by the Earth's rotation.

We find another problem with the time scale: the agreement of the General Conference of Weights and Measures to modify the civil time scale. Numerous proposals have been made, and we do not know which one will finally be adopted, so there is more uncertainty in preparing an astronomical calendar for the future.

This issue is particularly acute in the Chinese calendar, an astronomical calendar that needs to know the timing of the new Moons and winter solstices. We can determine these phenomena with great accuracy on the uniform time scale, but we need them on the civil time scale *. As we have said, the relationship between the two scales is uncertain.

In the current situation, we cannot be sure about preparing the Chinese calendar for the distant future. In our calculations, we use the UTC(2017) scale, which refers to the civil time standard at the beginning of 2017 (see Chapter 4); this means that the relationship between the uniform time scale and rotational time, or universal time, remains constant and is equal to the conversion that was in place at the start of 2017.

Types of calendars

We classify calendars into three types: lunar, lunisolar, and solar. A lunar calendar relies solely on the cycles of the Moon **, with months beginning on the new Moon ***. Typically, these calendars consist of 12 months, each lasting 29 or 30 days, resulting in approximately 354 days per year. Because of this structure, the dates of the seasons are not fixed and can vary within the lunar calendar.

Lunisolar calendars have lunar months, i.e. those that begin with the new Moon and can have 29 or 30 days, and the movement of the Sun determines which years are abundant or intercalary, i.e. those with 13 lunar months instead of 12. The result is that common years have 12 months and 354 days on average, while intercalary years have 13 months and 384 days on average #. The dates of the seasons are

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* Specifically, the time scale of calendars is civil time or Coordinated Universal Time (UTC), but the difference between universal time and UTC cannot exceed 0.9 seconds; therefore, for use in the calendar, we can consider UTC equivalent to universal time.

** The lunar movement used in calendars is synodic, that is, it is the movement in relation to the Sun, which determines the lunar phases.

***Hemerology sometimes defines the new Moon as the moment when the first crescent Moon is observed near the western horizon shortly after sunset. However, it is more common to understand the new Moon as the apparent geocentric conjunction of the Sun and the Moon, as described in Chapter 5, as does the Chinese calendar.

The term "leap" typically refers to the thirteenth month of abundant years and to years containing thirteen months. However, we will avoid using this term in this book as we believe it is inappropriate. The term originates from English, which refers to leap days as the *bissextum* days in the Gregorian calendar. The term leap means jump because, in years with

mobile in a lunisolar calendar, but they do not wander throughout the year; instead, their mobility is limited to an interval of approximately one month.

Finally, there are solar calendars, which completely ignore the Moon and are regulated only by the Sun's apparent movement around the Earth. They have a duration close to the tropical year, so the seasons remain practically fixed on the calendar, with a slight oscillation of a few days.

Another way of classifying calendars is by the methods used in their preparation. Observational calendars use astronomical observation to regulate the calendar; this is what happens with the Islamic calendar, where the beginning of the month is determined by observing the first crescent Moon. Astronomical calendars require astronomical calculations for their preparation, either of the Sun, the Moon, or both; this is the case of the Chinese calendar, where it is necessary to calculate the times of the new Moon and the winter solstices using astronomical theory, data that are used to prepare the calendar.

Finally, computational calendars use mathematical rules deduced from the mean motions of the Sun, the Moon, or both to determine the calendar. The Gregorian calendar is an example of this type of calendar.

The mean motions of the Sun and Moon used in computational calendars do not match astronomical values, leading to a gradual increase in errors over time. As a result, these computational calendars require corrections. In contrast, astronomical calendars are consistently aligned with actual astronomical data, so they do not need any corrections.

Some calendars cannot be classified in the above categories. One such calendar is based on Julian days, an uninterrupted count of days from a date far back in time.

Calendar meridian

In astronomical calendars, we calculate the time when certain astronomical phenomena occur, such as new Moons and winter solstices. However, the time when these phenomena occur has to be expressed in the civil time of some meridian, which may or may not be the Greenwich meridian. Therefore, an astronomical calendar is associated with a terrestrial meridian, or more precisely, it is associated with a time zone.

In the Chinese calendar, the new Moons and winter solstices are expressed in the civil time of the 120^{o} east time zone, where Beijing is located. This time zone is 8 hours ahead of civil time in Greenwich.

It is important to note that while the Chinese calendar is based on the Beijing time zone, the day reflects the time zone of the region where it is being used; this means that the Chinese lunar month starts on the day of the new Moon according to Beijing time. However, this does not guarantee that the month will begin with the new Moon in other time zones.

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366 days, it is necessary to proceed with a jump of the dominical letter. However, the dominical letter (a letter from A to G associated with each day) has no relation to the intercalary months. As usual, we will keep the term leap for the *bissextum* days of the Gregorian calendar but not for the lunisolar calendars.

The festivals

Religious festivals are fixed in the calendar, which is why religions have usually been at the origin of calendars. This is particularly true of the Gregorian calendar, which was developed by a papals commission at the end of the 16th century.

However, civil holidays are also fixed in the calendar, as are Chinese holidays, such as the Spring Festival or *Qingming*.

Traditional festivals are usually associated with dates on the traditional calendar, which is usually not the official calendar of each country. For this reason, it is common to have to use two calendars, the Gregorian for civil purposes and the traditional one to maintain the dates of the festivities; this happens in China and other countries in the region, which use the Gregorian calendar in daily life but set their festivals on the Chinese lunisolar calendar.

This also occurs to a certain extent in Christian countries, which use the Gregorian solar calendar for civil purposes but the Gregorian lunisolar calendar for establishing some religious festivals, such as Easter.

3.- Astronomical concepts

Ecliptic and celestial equator

The planet Earth is gravitationally linked to the Moon. Therefore, the Earth-Moon system revolves around the Sun. Specifically, the barycenter * of both stars describes the orbit.

We call the ecliptic the plane where the Earth-Moon barycenter orbit is, precisely defined as the plane containing the centre of the Sun, the Earth-Moon barycenter, and its velocity vector (that is, its movement)

Celestial coordinates are referred to the centre of the Earth; that is, they are geocentric. For this reason, we consider the geocentric ecliptic, a plane parallel to the ecliptic (as we have previously defined it) that passes through the Earth's centre. This plane is the reference for determining the geocentric ecliptic coordinates. Note that the Sun's centre is not usually located in the plane of the geocentric ecliptic (that is, it can have ecliptic latitude). In what follows, we will understand the ecliptic as the geocentric ecliptic.

Unless otherwise stated, we will assume that the Earth is the reference for describing celestial movements. Therefore, the Sun and the other stars in the firmament revolve around the Earth; that is, we consider the apparent movements of the Sun and other stars relative to the Earth.

The other fundamental plane in astronomy is the equator, which contains the plane of the terrestrial equator or great circle that divides the Earth into two hemispheres; likewise, the celestial equator divides the celestial sphere into two hemispheres **. Extending the axis of rotation of the Earth cuts the celestial sphere in two points: the north and south celestial poles, associated with the similar terrestrial poles.

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* The barycenter or centre of mass is a point on the line joining the centres of the Earth and the Moon, whose distance from the centre of the Earth is

$$r_B = \frac{m_M}{m_M + m_E} r_{E-M} = 4\ 672.83 \text{ km}$$

r_B is the distance from the centre of the Earth to the barycenter, r_{E-M} is the Earth-Moon distance, and m_E and m_M are the masses of the Earth and the Moon. Note that the barycenter is located inside the Earth, which has a radius of 6,370 kilometres.

** The celestial sphere is an imaginary sphere of indefinite radius with the Earth as its centre (either the centre of the planet or the observation point on the Earth's surface) and onto which all the firmament stars are projected.

The ecliptic is inclined with respect to the equator, forming an angle of approximately 23.5 degrees, which has secular and periodic variation.

The nodes are the points of intersection of the ecliptic curve * with the plane of the equator and are located on the line formed by the intersection of the planes of the equator and the ecliptic or line of nodes. We call the ascending node the point where the Sun's apparent orbit passes from the southern celestial hemisphere to the northern one and the descending node the point of the orbit where the Sun's apparent motion passes from the northern hemisphere to the southern one.

We call the ascending node the equinox because it is the position occupied by the Sun at the spring equinox or moment of the beginning of the astronomical spring. As we will see later, it is the point that, together with the ecliptic, allows us to define the celestial coordinates.

Precession and nutation

The ecliptic and the equator planes are moved by perturbations exerted on the Earth by the Sun, the Moon and the planets. The true equator is affected by the precession movement, which consists of a movement of the Earth's axis of rotation, which describes a conical movement and has a rotation period of approximately 26,000 years. As a consequence of the precession movement, the north celestial pole undergoes a rotation movement, such that over time, it is located at different points on the celestial sphere. Precession is the result of the combined action of the Sun and the Moon on the equatorial bulge of the Earth, hence the name lunisolar precession by which it is known.

The celestial equator (and the celestial poles) are also affected by nutation. While the celestial pole rotates by precession, it simultaneously describes a small elliptical motion (nutation ellipse), whose semi-axes take the values 9.21 and 6.86 arc seconds and has a period of approximately 18.6 years (very close to the time taken by the lunar nodes to make a complete revolution around the ecliptic).

As a result of both movements (precession and nutation), the celestial pole describes an undulating orbit composed of a secular movement and a periodic movement. The first is called precession, and the second is called nutation. The difference between the two is purely conventional. The term precession is reserved for secular or very long-period variations, and nutation is reserved for short-period terms.

The plane of the ecliptic is influenced by planetary perturbations. We must distinguish between two types of perturbations. One is a secular variation of the ecliptic and the other is a short-period variation produced mainly by the action of Venus and Jupiter. The latter is similar to the nutation is for the equator. The secular variation is called planetary precession, which is of much smaller value than the lunisolar precession.

The combined action of lunisolar and planetary precession is called general precession and has an approximate value of 50.3 arc seconds each solar year, approximately a period of 26,000 years.

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* We project the ecliptic plane onto the celestial sphere, resulting in a curve called the ecliptic or ecliptic curve.

Apparnet movements

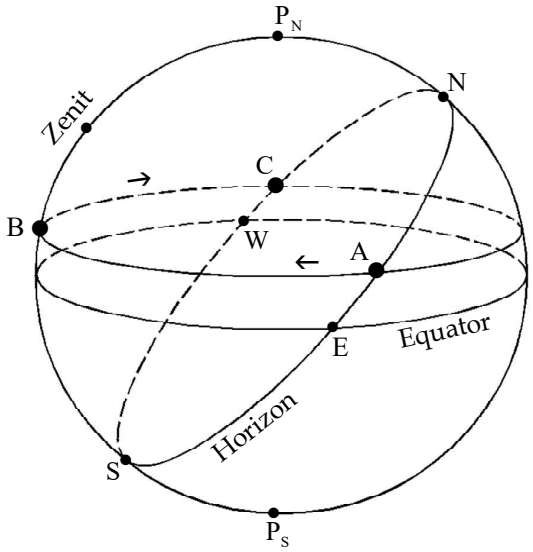
When observed from the northern hemisphere, the Earth rotates counter-clockwise around the Sun. The Earth's rotation on its axis and the Moon's rotation around the Earth have the same direction.

As a result of the Earth's motion around the Sun, the Sun appears to move counter-clockwise relative to the Earth or direct motion or prograde when observed from the north, corresponding to a west-south-east motion when viewed from the Earth's surface. As a result of the Moon's motion, it is observed from Earth to rotate with direct motion. Due to the Earth's rotation around its axis, the stars have a daily movement for the terrestrial observer that is east-south-west or retrograde.

Due to precession, the true equinox acquires a movement in the same direction as the diurnal movement of the celestial sphere, which means a movement celestial sphere's diurnal movement, which is contrary to the proper movement of the Sun, the Moon and the planets.

In summary, the Moon and the Sun have two movements: the daily one, which is caused by the rotation of the Earth on its axis and has a retrograde motion, and the proper movement, which is the result of the movement of the Earth around the Sun and of the Moon around the Earth, both of which have a direct motion.

Illustration 1.3 represents the celestial sphere, the equator and the horizon plane for an observer located at a point on the surface of the Earth's northern hemi-



Ilustration 1.3.- **DAILY MOVEMENT**. We represent the celestial sphere, divided into two hemispheres by the equator. The plane of the horizon of the observation site is inclined with respect to the equator, with the four cardinal points indicated: N, E, S and W. The north and south celestial poles are P_N and P_S. A celestial body, for example, the Sun, moves daily along the celestial parallel ABC in a retrograde direction, E-S-W or clockwise, when observed from the north celestial pole.

At point A, the Sun rises in the eastern zone. Then, the Sun heads south, increasing its altitude above the horizon until reaching point B, crossing the meridian of the observation site (the great circle where the zenith and the celestial poles are located), where it reaches its maximum altitude above the horizon. Later, the Sun begins to decrease its altitude until reaching point C, where the sunset takes place.

sphere. P_N is the north celestial pole (projection of the Earth's north pole on the celestial sphere), and P_s is the south celestial pole. N, E, S, and W are the cardinal points north, east, south, and west, which we represent on the horizon plane.

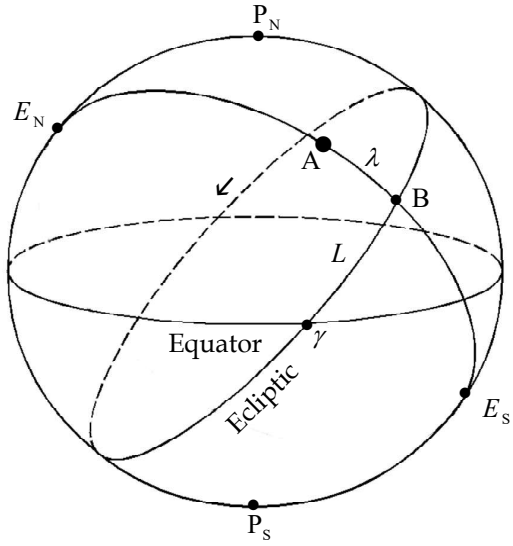
The daily movement of the stars, for example, the Sun, is carried out along a celestial parallel, that is, a minor circle parallel to the equator. Illustration 1.3 shows the apparent daily movement of the Sun. Point A corresponds to the place of sunrise. As time passes, the Sun moves in a retrograde direction, that is, E-S-W, gradually increasing its altitude above the horizon until it reaches point B located on the meridian of the place (that is, the great circle that passes through the zenith and the southern cardinal point) where it reaches its maximum altitude above the horizon. Later, the Sun descends, approaching the horizon that crosses it at point C, where sunset occurs.

Ecliptic coordinates

The plane of the ecliptic and the ascending node of the solar orbit (or equinox) form the reference system for defining the ecliptic coordinates.

The geocentric and geometric ecliptic longitude of a star is the angle measured on the ecliptic in the direct sense from the mean equinox of the date to the point of intersection of the ecliptic with the great circle that passes through the ecliptic poles and the star (circle of ecliptic latitude), which is measured from 0 to 360 degrees.

The ecliptic longitude, as we have defined it, is related to the mean equinox;



Ilustration 2.3.- **ECLIPTIC COORDINATES**. We represent the celestial sphere and draw the equator and the ecliptic, intersecting at point γ (ascending node, spring equinox or simply the equinox). We draw the north celestial pole P_N, the south celestial pole P_S and the north ecliptic poles E_N and south E_S. Point A is the position of a star, through which a circle of ecliptic latitude passes (a great circle that passes through A and the ecliptic poles). The ecliptic longitude L is the angle from γ to point B (intersection of the ecliptic with the circle of ecliptic latitude of star A), measured in the direct sense, that is, counterclockwise as seen from the north celestial pole. The ecliptic latitude λ is the angle between B and A, which is positive if the star is in the northern hemisphere and negative if it is in the southern hemisphere.

that is to say, the equinox is only affected by precession but not by nutation (in which case we would have the true equinox). The maximum value of the longitude correction by nutation is approximately 18.5 arc seconds. Nutation also affects the obliquity of the ecliptic, that is, the angle between the planes of the ecliptic and the equator, and can reach a maximum value of approximately 9.2 seconds. The nutation corrections can increase and decrease the ecliptic longitude and obliquity.

We have defined the geometric ecliptic longitude, which refers to the position occupied by the star at the time considered. Due to the finite speed of light, from Earth we observe the position of the star some time earlier, at the moment when the light left the Sun or the Moon (apparent and not geometric position). Then, from the geometric longitude, we must subtract the angle travelled by the star during the travel time of the emitted light to the observer, an angle called aberration, which in the case of the Sun is, on average, 20.5 arc seconds *.

We make the nutation and aberration corrections to find the apparent ecliptic longitude. Unless otherwise stated, the ecliptic longitudes we will consider are the geocentric and apparent longitudes of the Sun's or the Moon's centres.

The ecliptic latitude is the angle measured on the ecliptic meridian from the ecliptic to where the star is located. It has a positive value if the star is located in the northern hemisphere and a negative value if it is located in the southern hemisphere. Note that the Sun has a certain ecliptic latitude, sometimes positive and sometimes negative.

Solstices and equinoxes

In the Sun's apparent orbit (which practically coincides with the ecliptic curve), we can distinguish four points separated by an ecliptic longitude of 90 degrees (Ilustration 3.3). We start from the ascending node or spring equinox, which has an apparent longitude of 0^o and is one of the two points of intersection of the ecliptic and the equator. In its annual movement, the Sun rises in the northern celestial hemisphere until it reaches the point of greatest altitude above the equator (that is, the point of greatest positive declination), which corresponds to the summer solstice when the apparent longitude of the Sun is 90^o.

Continuing its annual movement, the Sun approaches the southern hemisphere, cutting the equator at the descending node or autumn equinox when its apparent longitude is 180^o. Finally, the Sun moves in the southern hemisphere until it reaches the point furthest from the equator (or of greatest negative declination), which occurs when its apparent longitude is 270^o, which corresponds to the winter solstice. From here, the same sequence is repeated.

Elliptical orbits

We assume that the orbits of the Sun and the Moon are ellipses continually modified by the perturbations of the other stars in the solar system.

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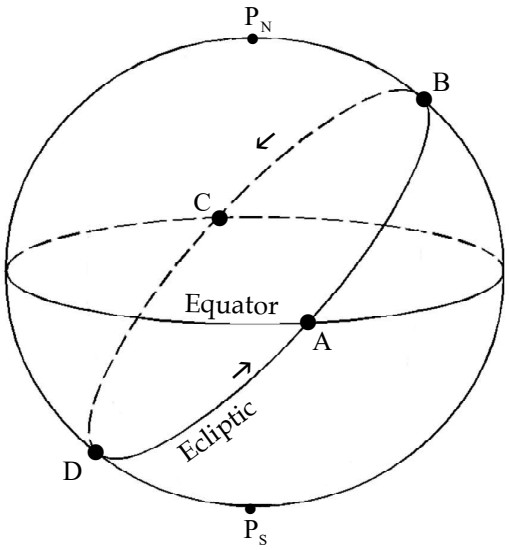
* We distinguish between aberration and light-time. We apply the first correction when the observer is in motion, while the second is when the star emitting the light is in motion. The Sun's longitude must be corrected for aberration, and the Moon's longitude must be corrected for light-time. In the text, we do not distinguish between one and the other phenomenon because, quantitatively, they are determined by the same formulas.



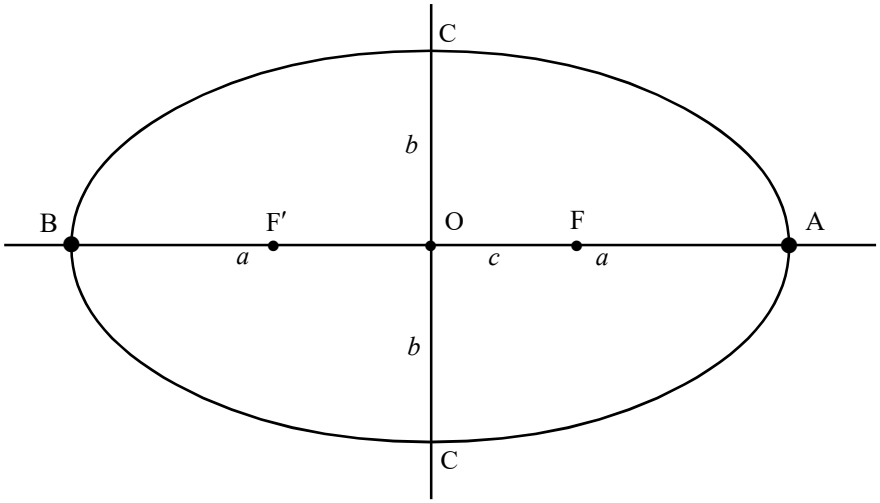
Ilustration 3.3.- **SOLSTICES AND EQUINOXES**. In the Sun's apparent orbit there are four singular points, which correspond to the apparent geocentric longitudes of 0, 90, 180 and 270 degrees and are marked in the drawing with the letters A, B, C, and D, respectively. The Sun describes an orbit annually, very close to the ecliptic curve; as we have said, both curves are slightly different due to the solar latitude. When the Sun passes through the ascending node A, the astronomical spring begins, and then the Sun has an apparent longitude of 0º. Later, the Sun rises above the equator until point B, with an apparent longitude of 90º, corresponding to the summer solstice when the astronomical summer begins. The Sun continues moving in relation to the Earth and reaches point C, the descending node or place of the autumn equinox, where the apparent longitude is 180º. Finally, when astronomical winter begins, the Sun passes through point D or the winter solstice. From then on, the sequence of equinoxes and solstices is repeated.

An ellipse is defined by the semi-major axis *a* and the eccentricity *e*. The first parameter determines the size of the ellipse, and the second tells us how flattened it is. The ellipses of the Sun and the Moon are close to the circumference; that is, their eccentricities are close to 0. The eccentricity of the Sun is approximately 0.0167, and that of the Moon ranges between 0.045 and 0.065.

According to Kepler's first law, the Earth is located at one of the two foci of the ellipse (in the case that we consider the movements of the Sun and the Moon relative to the Earth. The point of the ellipse closest to the focus is called the pericenter, taking the name of perihelion if it is the Sun that orbits it or perigee if it is the Moon. The point furthest from the focus is the apocenter, or aphelion if it is the Sun and apogee if it is the Moon.

The elliptical orbit lies in a plane inclined to the equator (and also to the ecliptic). It cuts the ecliptic plane at two points called nodes, the ascending and descending nodes. The ecliptic longitude of the node, together with the inclination of the orbital plane, defines the position of the orbital plane.

As we will see, the elliptical nature of the orbits of the Sun and the Moon influences the Chinese calendar, as a consequence of Kepler's second law, which establishes that the velocity of the orbiting star depends on its proximity to the fo-



Ilustration 4.3.- **ELLIPTICAL ORBIT.** We represent an elliptical orbit. $a = \mathrm{OA} = \mathrm{OB}$ is the major semi-axis, $b = \mathrm{OC}$ is the minor semi-axis and $c = \mathrm{OF} = \mathrm{OF}'$ is the focal length. F and F′ are the two foci. The eccentricity of the orbit is $e = c / a$. If we assume that the Earth is at F, point A is the pericentre at distance $a(1-e)$ from the focus; and point B is the apocentre at distance $a(1+e)$ from the focus F.

cus, its velocity being greater when it is at the pericenter. We point out that the ellipses followed by the Sun and the Moon are altered by the action of other bodies in the solar system, the lunar ellipse being more perturbed than the solar one.

4.- Time scales

Astronomical calendars are regulated by astronomical phenomena, which in the Chinese calendar are the new Moons and the winter solstices. The time scale for measuring the movements of the Sun and the Moon is uniform time, which is the scale on which we calculate the dates of the new Moons and the winter solstices. However, calendars, including the Chinese, have as their essential elements days of 24 hours of civil time. In astronomical calendars, we use two time scales, civil and uniform.

As we will see below, civil time differs from astronomy's uniform time. Although there is a relationship between the two time scales, this relationship is variable; what is more, we cannot know in advance the relationship between the two time scales due to the non-uniformity of the Earth's rotation on its axis, which creates a significant problem for preparing the Chinese calendar in the future.

The problem cited is not important for the near future, but it does not provide security in preparing the calendar for the distant future. The matter is complicated by the intention to modify the civil time scale, but there is no certainty that a new one will be implemented or when it will be implemented, nor do we know what the new scale will be.

In this chapter, we explain the time scales related to the calendar, their relationship and the conversion methods between civil time and uniform time.

Universal time

The hour angle is an astronomical magnitude that allows us to know the daily movement of a star. We define it as the angle measured on the celestial equator from the upper meridian of the place (that is, the semi-meridian where the zenith of the place is) to the hour circle of the star (the greatest circle that passes through the celestial poles and the star). We measure the hour angle in the retrograde direction, that is, in the direction of the diurnal movement of the sky. The hour angle is measured in hours, understanding that 24 hours are equivalent to 360 degrees.

Sidereal time or equinox hour angle is an astronomical time scale. We distinguish between true and mean sidereal time, depending on whether we take the true equinox (with precession and nutation) or the mean equinox (with precession but without nutation) as a reference.

True solar time is, by definition, the hour angle of the true Sun. It is a non-uniform time due to the irregular motion of the Sun, nutation and the secular accelerations of the equinox and the Sun.

Mean solar time is the linear terms of true solar time. We define universal

time UT as mean solar time plus 12 hours, i.e. UT is zero when the mean Sun passes through the lower meridian (the semi-meridian where the nadir is) and 12 hours when it passes through the upper meridian. UT is the time referred to the Greenwich meridian, from which we deduce the time in another meridian.

The mean Sun is a fictitious body that moves along the equator, and its hour angle is mean solar time. The mean Sun has a uniform motion relative to the meridian but an accelerated motion relative to the equinox, caused by the equinox's precession rather than the Sun's acceleration.

With L_m, we represent the mean geometric ecliptic longitude of the Sun with respect to the mean equinox. It has no periodic terms, so it is a polynomial expression in function of time. L_m defines another fictitious Sun that travels along the ecliptic and considers the Sun's acceleration.

We identify the mean Sun's right ascension α_m with the mean longitude L_m, provided that we take apparent and not geometric values and that only the secular acceleration of the equinox appears and not the Sun's acceleration; then universal time is

$$\mathrm{UT} = T_m - \alpha_m + 12^h \tag{1.4}$$

T_m is the mean sidereal time. We find the true sidereal time by observing fundamental stars and the mean sidereal time by using the theory of nutation. Finally, by (1.4), we find the universal time.

The universal time found by an observatory is called UT0. If we correct for the movement of the Earth's pole * we find the scale UT1.

Definition of the second

The second, as a unit of time, was defined as 1/86,400 parts of a mean solar day, which is the time it takes the mean Sun to travel a complete circumference with respect to the meridian.

Knowing the non-uniformity of the Earth's rotation, which means that the mean Sun does not have a uniform motion, the ephemeris second was defined in 1960 based on the Sun's motion. The ephemeris second unit was used to define the ephemeris time scale, a uniform time scale used as the argument of the astronomical ephemeris.

Having proven the excellence of atomic clocks, the General Conference on Weights and Measures defined the atomic second in 1967 in terms of the quantum transition between the two hyperfine levels of the fundamental state of the cesium-133 atom.

International Atomic Time Scale

International Atomic Time (TAI) is a time scale based on the reading of a set of atomic clocks operating in various establishments. It uses the atomic second as its

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* The Earth's poles, or points on the Earth through which the imaginary axis of rotation passes, move relative to the Earth (this is different from the precession movement, which is a movement of the poles relative to the celestial sphere) and is known as Chandler's oscillation. It has a period of 433 days, with the instantaneous pole describing an irregular circumference with its centre at the mean pole, with a diameter that fluctuates between 3 and 15 metres.

unit and is measured on the surface of the Earth's geoid.

The TAI scale was made to coincide with UT1 on 1 January 1958 at 0 hours. At that time, the difference between UT1 and the ephemeris time was 32.184 seconds. Since TAI and the ephemeris time are uniform, they will maintain a constant difference of 32.184.

Terrestrial time TT is the local time of a clock at rest on the surface of the Earth's geoid. Its unit is the atomic second, and it is a uniform time. Terrestrial time coincides with the time of the ephemerides, which, for practical purposes, is the argument of the ephemerides of the stars. One realization of terrestrial time is through international atomic time.

$$TT = TAI + 32^{s}.184$$

relationship that remains unchanged.

Coordinated Universal Time Scale

Some time users, as physicists or engineers, need a uniform time scale, such as International Atomic Time. Other activities, such as astronomy or navigation, need Universal Time. The Coordinated Universal Time scale UTC was created to satisfy both users.

UTC is a mixture of atomic time and universal time. It follows the TAI scale and is, therefore, a uniform time, which has the atomic second as its unit. However, to avoid distancing itself from UT1, an extra or intercalary second is occasionally added, preventing the difference $|UTC - UT1|$ from exceeding 0.9 seconds due to the slowing of the Earth's rotation.

The International Earth Rotation Service (IERS) decides when to place a leap second after continuously monitoring the Earth's rotation on its axis and warning, normally six months in advance, of the possibility that UT1 might move away from UTC by more than 0.9 seconds. Leap seconds should preferably be placed at the end of December or June, as has been the case since 1972, the date of implementation of UTC.

It may happen that instead of adding a second to the UTC scale, it is necessary to subtract a second due to an increase in the velocity of the Earth's rotation. This circumstance has not occurred since the implementation of Coordinated Universal Time.

UTC was defined in 1960. It operates primarily with frequency adjustments and small steps to keep it as close as possible to universal time. The current UTC scale came into effect on 1 January 1972, when the difference between TAI and the old UTC scale was 10 seconds.

UTC is the TAI from which we subtract an integer number of seconds (the 10 seconds indicated above and the seconds that are intercalated)

$$UTC = TAI - D\mathit{t} = TAI - 10 - \mathit{int}$$

int is the net number of leap seconds. Until 2025, 27 leap seconds have been placed, so $\Delta t = 10 + \mathit{int} = 37$ seconds.

Empirical formula to find ΔT

Due to the Earth's irregular rotation, the terrestrial time scale does not keep pace with universal time. To study this discrepancy, we use the lengthening of the

day or *lod*, defined as the time that the length of the mean solar day exceeds 24 hours of uniform time or terrestrial time. The mean solar day always has 24 hours of universal time, but its length varies when measured in terrestrial time.

We define ΔT by

$$\Delta T = TT - UT1$$

expressed in seconds. With this definition we determine the lengthening of the day, which expressed in milliseconds per day is

$$lod = \frac{1}{36.525}\frac{d\Delta T}{dT} \tag{2.4}$$

T are Julian centuries of 36.525 days counted from the beginning of 2000. Analyzing old astronomical observations, Stephenson et al. (2016) found the following empirical fit to the data

$$lod = 3.115 + 1.78T - 4.0\sin\left(2\pi\frac{T+1.75}{15}\right),$$

the argument is in radians and *lod* in ms/day. Integrating (2.4) we find

$$\Delta T = \Delta T_0 + \int_{T_0}^{T} \left(36.525 lod\right) dT$$

T_0 corresponds to a reference year, which we take as 2017 when $\Delta T = 68.59$ seconds. Then

$$\Delta T = -193.629 + 32.507T^2 + 113.775T + 348.788\cos\left(2\pi\frac{T+1.75}{15}\right). \tag{3.4}$$

ΔT is related to leap seconds. UTC and UT1 differ by a maximum of 0.9 seconds, so they can be considered approximately equal. From the above definitions, we find

$$int \approx \Delta T - 42.184$$

int is the net number of leap seconds placed on the date to calculate ΔT.

Conversion from TT to UT

The astronomical phenomena that we use to prepare the Chinese calendar, such as new Moons and winter solstices, are calculated in terrestrial time, but they must be expressed in the Beijing time zone, that is, UTC+8 hours, for their use in the calendar.

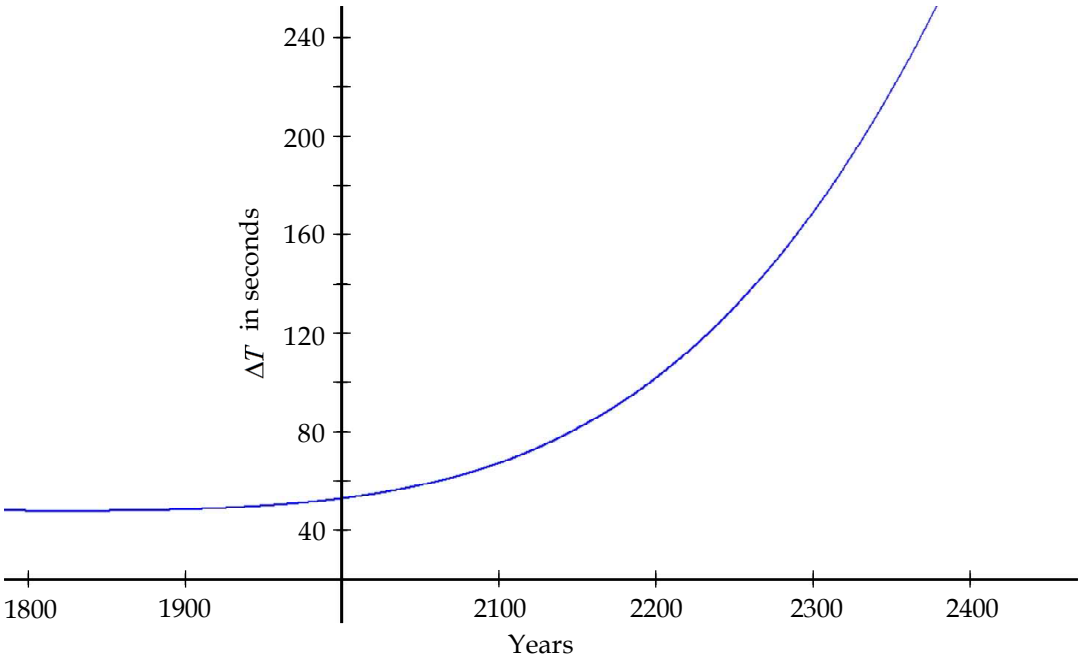
To find the moment of an astronomical phenomenon in universal time, the ΔT value for that moment must be subtracted from the time in TT.

On the other hand, the relationship between UTC and TT is

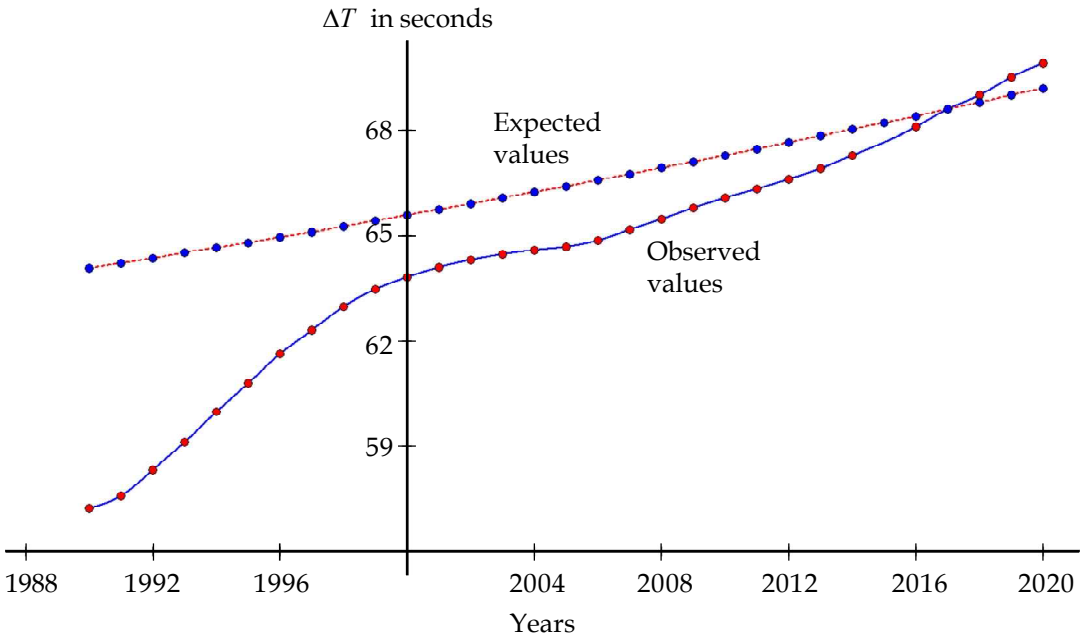
$$\text{UTC} = TT - 42.184 - int \approx TT - \Delta T \tag{4.4}$$

Since the difference between UTC and UT cannot exceed 0.9 seconds, we can consider them to be approximately equal. We must subtract ΔT from the date to move time from TT to UTC. 27 leap seconds were placed until 2024, the last one occurring on December 31, 2016, meaning that the Earth's rotation has been stabilized for 8 years. At the end of 2024, the difference between UTC and TT was 69.184 seconds.

The above refers to the transformation between time scales. Another matter is the transformation of periods, specifically the conversion of the lunation and the



Ilustration 1.4.- $\Delta T = TT - UT1$. We show the graph of the function (3.4) from 1800 to 2400. It shows parabolic growth, which means that leap seconds will become more frequent over time.



Ilustration 2.4.- $\Delta T = TT - UT1$. We show ΔT for the years 1988 to 2024, comparing the measured values with those deduced from formula (3.4). On the vertical axis is the value of ΔT in seconds, and on the horizontal axis are the years. On the upper curve are the values predicted by (3.4), which correspond approximately to a straight line in the period. On the lower curve are the observed values, which deviate slightly from those predicted. In 2017 both curves intersected since (3.4) was deduced, taking that moment as a reference.

year of the winter solstice from the TT scale, on which they are calculated, to the UTC civil time scale.

Let 1 and 2 be the moments when a period, for example, a lunation or a winter seasonal year, begins and ends. The interval in terrestrial time is $TT_2 - TT_1$, and the same interval in uniform time is $UT_2 - UT_1$, which are related by

$$\begin{aligned} UT_2 - UT_1 &= TT_2 - TT_1 - (\Delta T_2 - \Delta T_1) = TT_2 - TT_1 - \frac{d\Delta T}{dT}(T_2 - T_1) = \\ &= TT_2 - TT_1 - \frac{d\Delta T}{dTT}(TT_2 - TT_1) = (TT_2 - TT_1)\left(1 - \frac{d\Delta T}{dTT}\right), \end{aligned} \tag{5.4}$$

T are Julian centuries, and $T_2 - T_1$ is the period considered in Julian centuries. In (5.4), it must be considered that ΔT is the difference between two times (terrestrial and universal) expressed in different units, one in the atomic second and the other in the second deduced from the mean solar day. In (5.4), ΔT is expressed in days, as is TT, but in (3.4), it is expressed in seconds, so we have to make the conversion of (3.4) to express it in days

$$\Delta T = \left[-193.629 + 32.507T^2 + 113.775T + 348.788\cos\left(2\pi\frac{T+1.75}{15}\right)\right] \Big/ 86400 =$$

$$= -2.2411\cdot 10^{-3} + 3.7624\cdot 10^{-4}T^2 + 1.3168\cdot 10^{-3}T + 4.0369\cdot 10^{-3}\cos\left(2\pi\frac{T+1.75}{15}\right),$$

and

$$\frac{d\Delta T}{dTT} = \frac{1}{36525}\frac{d\Delta T}{dT} = 2.0602\cdot 10^{-8}T + 3.6052\cdot 10^{-8} - 4.6296\cdot 10^{-8}\sin\left(2\pi\frac{T+1.75}{15}\right),$$

calculated in day/day. By (5.4), we find the equation to transform periods TT into periods UT

$$UT_2 - UT_1 = (TT_2 - TT_1)\left[1 - 2.0602\cdot 10^{-8}T - 3.6052\cdot 10^{-8} + 4.6296\cdot 10^{-8}\sin\left(2\pi\frac{T+1.75}{15}\right)\right]. \tag{6.4}$$

For example, the synodic month or mean lunation expressed in units of terrestrial time is

$$m_s = 29^d.530\ 58885 + 2.163\cdot 10^{-7}T$$

applying (6.4) we find

$$m_s \approx 29^d.530\ 587\ 79 - 3.920\ 891\ 99\cdot 10^{-7}\,T - 1.367\ 148\ 14\cdot 10^{-6}\sin\left(2\pi\frac{T+1.75}{15}\right) \tag{7.4}$$

which is the synodic month in units of universal time, having neglected the smallest terms of the development. It is curious to observe that when the lunation is in terrestrial time, it increases with time. However, when we express the lunation in universal time, it decreases with time.

Problems caused by leap seconds in the Chinese calendar

The new Moon and the winter solstices regulate the Chinese calendar. The first phenomenon determines the beginning of the lunar months, and the second determines whether the year is common (12 lunar months) or embolismic (13 lunar months). We also need to know the movement of the Sun along the ecliptic, from

which we determine which month is embolismic or intercalary in the case of an abundant year.

We inevitably have to use the TT time scale to find the astronomical phenomena required by the calendar, and we also have to use the UTC civil time, as it is the scale on which the Chinese calendar must finally be given.

According to official rules for preparing the Chinese calendar, the times of the new Moons and winter solstices must be known with an accuracy of 1 second. Achieving this goal is impossible when we want to prepare the calendar for the future since we do not know when and how many leap seconds will be added to UTC.

Two problems can occur. One is that the time of the new Moon is very close to 0 hours, and since we do not know the UTC precisely, we cannot know for sure if the lunar month begins on one day or another. The other problem, more serious but rarer, can occur when the winter solstice is very close to 0 hours on the day of the beginning of the month. Then, we cannot know for sure if the 11th month, the one that regulates the calendar, is in one lunar month or the previous one.

Therefore, there is an uncertainty when calculating the Chinese calendar in the future since the exact relationship between TT and UTC is unknown. There are not many critical situations, the first of which will occur with the new Moon on September 29, 2057 (Table 5.8), which, according to our calculations, will occur at 0 hours, 1 minute, 48 seconds TT, referring to the Beijing time zone. From (3.4), we find $\Delta T = 86.4$ seconds, so in UT, the New Moon will be at 0 hours, 0 minute, 22 seconds very close to midnight.

Possible changes in the civil time scale

The current civil time system based on UTC and leap seconds has several problems, So there is speculation about the change of civil time, which would impact the preparation of the Chinese calendar.

The 27th General Conference on Weights and Measures held in November 2027 decided to abandon leap seconds on or before 2035. Faced with this challenge, the international scientific community can choose several options:

* Maintaining the *status quo*; this would preserve current problems, such as the adaptation of equipment when there is a leap second; the need to have an archive of leap seconds placed in the past, which will gradually become more extensive, which would increase the possibility of error (as shown in Illustration 1.11, leap seconds will become more and more frequent) or the emergence of new time scales to alleviate the difficulties, as has already occurred with the GPS system, which uses an adaptation of the UTC scale, which only takes into account 19 leap seconds and is not affected by new intercalations.
* Increase UTC leaps; instead of occasionally inserting a second, make leaps of several seconds or even one minute.
* Periodically adding a second to UTC, doing something similar to the intercalation of leap days in the Gregorian calendar, intercalating a second on pre-established periodic dates.
* Establish TAI as civil time. This type of change is difficult to implement, as all countries must collaborate to make it simultaneously.

* Disconnect leap seconds from the UTC scale. From a specific date onwards, stop adding leap seconds to UTC. This way, the difference with TAI will remain constant, regardless of the variation in the Earth's rotation.

Some astronomical calculations, such as the moments of sunrise and sunset or determining the position of a star above the horizon, require universal time. Therefore, regardless of what happens with civil time, we still need to know ΔT to convert from TT to UT and vice versa.

Given the uncertainty surrounding the future of the UTC scale, in this book, we will perform calculations on the UTC(2017)+8 scale, the coordinated universal time scale valid at the beginning of 2017 corrected for Beijing standard time.

Julian Day

The Julian period is a chronological dating technique introduced by Scaliger in 1582 and later applied to the computation of days by the astronomer William Herschel in 1851. Scaliger recorded the reasons for giving his cycle the name Julian: «I have called it the Julian period because it conforms to the Julian year.»

The Julian period is made up of three distinct cycles: the solar cycle, which lasts 28 years; the Meton cycle, spanning 19 years; and the indiction cycle, which lasts 15 years. When we multiply the lengths of these three cycles, we arrive at the total duration of the Julian period, which is 7,890 years. In summary, the Julian period is the product of the solar cycle (28 years), the Meton cycle (19 years), and the indiction cycle (15 years), leading to a total of 7,890 years.

The origins of the three periods mentioned are that year 1 of the Christian era corresponds to year 2 of the Meton cycle, year 10 of the solar cycle, and year 4 of the indiction. Consequently, the beginnings of these three cycles coincide in the year - 4712 (or 4713 BCE), marking the origin of the first Julian period. The second cycle will commence in the year 3178.

The date of a calendar is expressed in a heterogeneous way, consisting of year, month, day, hour, minute, and seconds, so it is not suitable as a temporal argument. To avoid this inconvenience, in astronomy we use the Julian Day, which is an uninterrupted count of days from the distant past. Thus, using a single decimal number, we can express the date and time of an event.

The count of astronomical Julian Days begins on January 1, 4713 BCE of the Julian calendar at 12 hours of terrestrial time (or another time scale), although the chronological Julian Day begins at the same time as the civil day, that is, at midnight; that is, at 0 hours on January 1, 4713 BCE.

The Julian Day count starts at 0, meaning January 1, 4713 BCE, is designated as Julian Day 0. The chronological Julian Day is always represented as an integer, while the astronomical Julian Day can have decimal values to indicate fractions of the day. In the following discussion, we will focus on the astronomical Julian Day, which is counted from noon rather than midnight, following the terrestrial time scale.

The Julian year has a mean length of 365.25 days, which means that a Julian century consists of 36,525 days. For calculating astronomical ephemerides, the number of Julian centuries is determined from the epoch J2000.0, which starts at 12 hours on January 1, 2000. To find the number of Julian centuries that have elapsed until a given Julian Day (*JD*), we use the following calculation

$$T = \frac{JD - 2\,451\,545}{36\,525}$$

2,451,545 is the Julian Day of epoch J2000.0.

Appendix III explains the algorithm used to convert a date in the Gregorian calendar to its corresponding Julian Day and vice versa. This conversion is straightforward with the Gregorian calendar, as it is a computational calendar. However, the process is more complex when working with the Chinese calendar.

5.- Movements of the Sun and the Moon

Keplerian orbits

The Sun's apparent motion follows approximately an orbit that obeys Kepler's laws and is perturbed by the action of other objects in the solar system. To a lesser extent, the lunar motion can be considered as Keplerian, although strongly perturbed mainly by the action of the Sun.

To determine the position of a object in an undisturbed Keplerian orbit, we need seven data. Two data help us determine the orbital plane: the longitude of the ascending node or angle measured in direct sense on the ecliptic between the equinox γ and the ascending node (point of intersection of the orbit with the ecliptic); and the inclination i of the orbit with respect to the ecliptic.

The determination of the ellipse that describes the object requires two data: the semi-major axis a and the eccentricity e. We define the position of the ellipse in the orbital plane by the latitude argument of the perigee ω, or angle between the ascending node and the pericenter, measured in the plane of the orbit.

As an alternative to ω, we use the longitude of the perigee $\overline{\omega}$, or angle measured in two planes: from the equinox γ to the ascending node and from this point to the pericenter, that is

$$\overline{\omega} = \Omega + \omega.$$

To know the motion of the object as a function of the time is necessary to know the mean motion n with respect to perigee, that is, 360^{o} (or 2π radians) divided by the time between two consecutive passages of the object through the perigee. And finally, it is necessary to know at some moment the position of the object that orbits.

Movement of the Sun

We assume that the Sun's apparent motion around the Earth follows an ellipse whose orbital parameters (eccentricity and longitude of the perihelion) vary secularly over time due to perturbations from other celestial bodies in the solar system.

We consider that the VSOP82 theory gives the geometric mean ecliptic longitude (without aberration) of the Sun with respect to the mean equinox of the date (that is, without considering nutation but considering precession).

$$L_m = 280^{o}.466\ 448\ 514 + 36\ 000.769\ 8231\ T + 0.000\ 303\ 678\ T^2 + 0.000\ 000\ 0212\ T^3 \quad (1.5)$$

T are the Julian centuries (36,525 days long) counted from the beginning of the year 2000.

The mean longitude of the perihelion varies according to the expression

$$\bar{\omega}_m = 282^o.937\ 348 + 1.719\ 5269\ T + 0.000\ 459\ 62\ T^2 + 0.000\ 000\ 499\ T^3 \qquad (2.5)$$

and the mean eccentricity of the orbit has the secular variation

$$e = 0.016\ 708\ 62 - 0.000\ 042\ 037\ T - 0.000\ 000\ 1236\ T^2 + 0.000\ 000\ 000\ 04\ T^3 .$$

The mean anomaly M is the angle between the mean perihelion and the mean position of the Sun which has the longitude (1.5)

$$M = 360 + L_m - \bar{\omega}_m = 357^o.529\ 101 + 35\ 999.050\ 296\ T - 0.000\ 1559\ T^2 . \qquad (3.5)$$

The true anomaly θ is the angle from the mean perihelion to the true Sun measured in the direction of solar motion (direct motion) and satisfies the relationship

$$\theta = L + 360 - \bar{\omega}_m \qquad (4.5)$$

where L is the longitude of the true Sun, i.e. the mean longitude (1.5) plus the periodic terms of the solar motion. The eccentric anomaly E is determined by the Kepler equation

$$E - e \sin E = M$$

and the true anomaly by the equation

$$\tan\frac{\theta}{2} = \sqrt{\frac{1+e}{1-e}}\tan\frac{E}{2}.$$

From the two equations above we find the true anomaly as a function of the mean anomaly, which is called the center equation; by (4.5) we find

$$L = L_m + \left(2e - e^3/4\right)\sin M + 5/4\,e^2 \sin 2M + 13/12\,e^3 \sin 3M + ... \qquad (5.5)$$

which is the true longitude of the Sun if Kepler's laws are followed, taking into account the secular variation of the eccentricity and the longitude of the perihelion.

As we have said, the Sun's apparent motion is not entirely Keplerian as a result of the perturbations of the other bodies of the solar system, which would be represented by periodic terms that would have to be added to (5.5). Nevertheless, (5.5) represents the Sun's apparent motion with a good approximation and will be useful for calculating average values, for which the only periodic terms to consider are those of the center equation. In this approximation, we can consider the ecliptic latitude to be zero and, therefore, assume that the Sun's centre is always in the ecliptic plane.

Note that (5.5) is the geometric longitude, i.e. it does not consider aberration or nutation. We can consider the average aberration, i.e. independent of distance, so from (5.5) we would have to subtract the aberration constant of 20.49552 arc seconds. In Table 1.5 we show the results found by (5.5) corrected for aberration and those obtained by the VSOP87 solar motion theory, where we confirm that the Sun's apparent motion is very close to the Keplerian motion.

VSOP87 theory simplified

We can use (5.5) for an approximate solar longitude calculation, which gives a maximum error of approximately half an arc minute. This error corresponds to about 12 minutes of time, an excessive error for calculating the Chinese calendar.

In order to achieve a smaller error in calculating the ecliptic longitude, we will use in this book the simplified version of the VSOP87 theory of Bretagnon and Simon proposed by Meuss. In its extended version, the theory gives a precision of 0.01

Date	Hour	Longitude (Keplerian theory)	Longitude (VSOP87 theory)	Difference (seconds)
2000/01/01	00:00:00	279-52-01	279-51-30	-29
2000/07/01	12:00:00	99-56-59	99-56-36	23
2010/01/01	00:00:00	280-27-01	280-27-02	-1
2010/07/01	12:00:00	99-33-19	99-33-11	8
2020/01/01	00:00:00	280-00-52	280-00-32	20
2020/07/01	12:00:00	100-06-36	100-06-19	17

Table 1.5.- **ECLIPTIC LONGITUDE OF THE SUN**. In the table, we calculate the Sun's ecliptic longitude using the Keplerian theory (5.5) corrected by the aberration constant and the longitude derived from the VSOP87 theory simplified by Meuss. We find that the values obtained are very close, with a difference of a few arc seconds, which means that the Sun's motion is approximately Keplerian, as long as we consider the secular variation of the eccentricity and the precession of the perihelion.

arcseconds, and in the simplified version, 1 arcsecond. This precision is sufficient for calculating the Chinese calendar, and we will use it in what follows.

In the VSOP87 theory, the ecliptic longitude in radians has the form

$$L = \left(L_0 + L_1\tau + L_2\tau^2 + L_3\tau^3 + L_4\tau^4 + L_5\tau^5 \right) \Big/ 10^8 \qquad (6.5)$$

τ is the Julian millennium of 365,250 days elapsed since the beginning of the year 2000. Each term is of the form

$$A\cos\left(B + C\tau\right) \qquad (7.5)$$

A, B, and C are parameters found in the Appendix II. Using formulas like (6.5) and (7.5), we determine the ecliptic latitude and the Earth-Sun distance.

A highly accurate method for calculating the positions of the Moon and the Sun is to utilize the Jet Propulsion Laboratory Development Ephemeris (JPL-DE) model. This model is created through numerical integration of the equations of motion, which are adjusted on observational data. It takes into account gravitational influences from 343 asteroids, the effects of General Relativity, and accelerations resulting from the non-sphericity of the Earth, Moon, and Sun. The Purple Mountain Observatory, which manages the Chinese calendar, uses the JPL-DE algorithm to determine the positions of the Sun and Moon.

Solstices and equinoxes

The spring equinox is defined as the moment when the centre of the Sun has an apparent ecliptic longitude (considering aberration and nutation) of 0 degrees. The summer solstice occurs when the longitude is 90º; at the autumn equinox, the apparent longitude of the Sun is 180º; and the winter solstice occurs when the longitude is 270º. In this section, we focus on the winter solstice because it is the seasonal point that regulates the Chinese calendar.

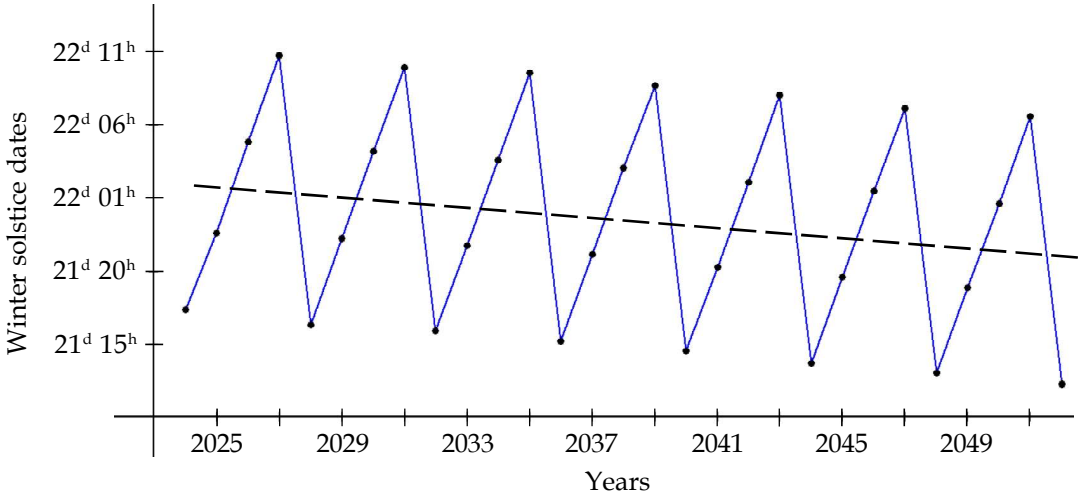
Date and time of the winter solstice		Differences
2024/12/21	17:20:27	
2025/12/21	23:02:50	05:42:23
2026/12/22	04:50:09	05:47:19
2027/12/22	10:42:01	05:51:52
2028/12/21	16:19:36	-18:22:25
2029/12/21	22:13:54	05:54:18
2030/12/22	04:09:31	05:55:37
2031/12/22	09:55:19	05:45:48
2032/12/21	15:55:48	-17:59:31
2033/12/21	21:45:43	05:50:24
2034/12/22	03:34:00	05:48:17
2035/12/22	09:30:50	05:56:50
2036/12/21	15:12:52	-18:17:58
2037/12/21	21:07:35	05:54:43
2038/12/22	03:02:22	05:54:47
2039/12/22	08:40:31	05:38:09
2040/12/21	14:32:50	-18:07:41

Table 2.5.- **DATES OF THE TRUE WINTER SOLSTICE**. The table shows the dates and times of the winter solstice for several years of the Gregorian calendar. The time is expressed in UTC and the Beijing time zone (UTC+8 hours). In the third column is the time difference between two consecutive solstices. We can see that in each non-leap year, the solstice advances by a little less than 6 hours; it comes approximately a quarter of a day later in the Gregorian calendar. In a leap year, the winter solstice comes approximately 18 hours earlier, which allows the three advances that occur in the three non-leap years to be counteracted. However, the delays and advances of the winter solstice as a result of the intercalation of leap years do not exactly cancel each other out; for this reason, it is necessary to correct the rule of intercalation of leap years (called the Gregorian rule) in order to stabilize the seasons. We have made the calculations with the simplified VSOP87 theory.

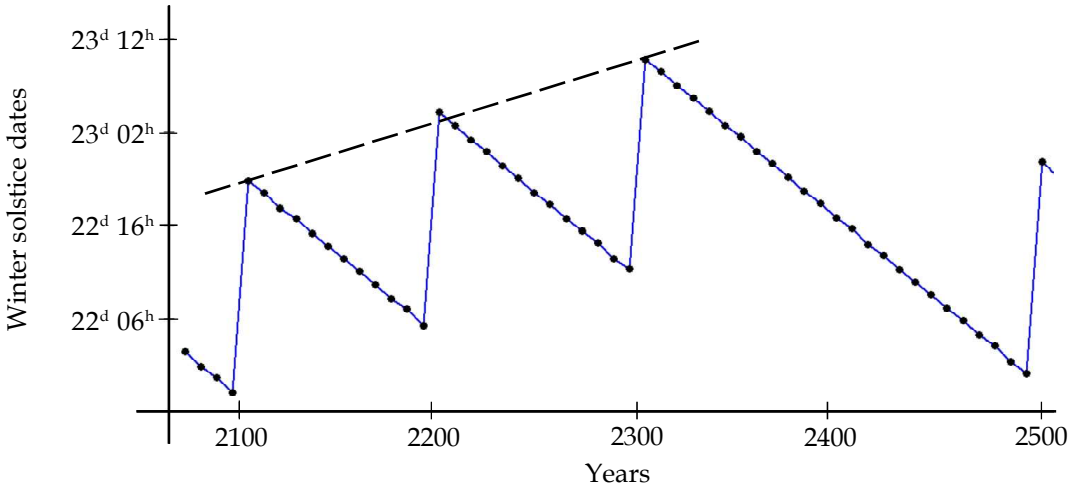
Table 2.5 shows the Gregorian dates of the winter solstices from 2024 to 2040 using Beijing time zone time. We find that the winter solstice days can be December 21, 22, or 23.

Illustration 1.5 shows how the winter solstice dates move forward and backwards. In each non-leap year, the solstice arrives approximately 6 hours later. In leap years, the solstice arrives approximately 18 hours earlier than in the previous year, which corrects for previous shifts.

We see in Illustration 1.5 that there is a gradual tendency for winter solstices to occur earlier and earlier in the Gregorian calendar. If no remedy were found, the



Ilustration 1.5.- **DATES OF THE WINTER SOLSTICE IN THE GREGORIAN CALENDAR**. The vertical axis shows the Gregorian date of the winter solstice in the Beijing time zone, and the horizontal axis shows the years. In each non-leap year of the Gregorian calendar, the winter solstice occurs approximately 6 hours later than the previous year. In a leap year (one that is divisible by four), the winter solstice occurs 18 hours earlier, counteracting the advances that occurred in non-leap years. The dashed line shows that the winter solstices gradually arrive earlier and earlier according to the calendar; this means an excess of leap years. This drift would cause the winter solstice and the other seasonal points to shift on the calendar. The Gregorian rule of leap years is applied to avoid this shift. According to this rule, every 400 years, 3 leap years are removed, which prevents the winter solstices from shifting permanently on the Gregorian calendar.



Ilustration 2.5.- **GREGORIAN RULE OF LEAP YEARS.** The dots correspond to winter solstices of the year preceding a year divisible by 4. The years 2100, 2200 and 2300 will not be leap years although they are divisible by 4, this causes the date of the winter solstice to be approximately one day later, thereby counteracting the gradual advancing of the solstice date that we saw in Illustration 1.5. By removing the leap years of centennial years, the date of the solstice begins to be later (dashed straight line), which is corrected by making the year 2400 a leap year.

solstice would continue to shift on the calendar. We avoid this drift by using the Gregorian rule of intercalation of leap years, according to which every 400 years, 3 leap years are removed; this allows the date of the winter solstice and the other seasonal points to be stabilized in the calendar.

In Illustration 2.5 we show how the Gregorian rule for intercalation works. 2100 will not be a leap year, preventing the tendency for the solstice to be earlier and earlier. The years 2200 and 2300 will not be leap years either; if we followed this rule, the winter solstice date would gradually be delayed, as shown by the broken straight line in Illustration 2.5. To prevent this tendency, the year 2400 will not be a leap year, which will cause the date of the solstice to be earlier, counteracting the delays caused by eliminating the leap day in the three previous centennial years.

Tropical year and seasonal years

We define the tropical year as the time that has to pass so that the mean longitude of the Sun L_m referred to the mean equinox of the date increases by 360 degrees. From (1.5) we find that the tropical year is

$$a_T = 365^{\mathrm{d}}.242\,189\,67 - 0.000\,006\,1619\,T - 6^{\mathrm{d}}.4514 \cdot 10^{-10}\,T^2,$$

expressed in Terrestrial Time TT; we see that every Julian century, the tropical year decreases by 0.53 seconds.

Other definitions of the year exist depending on the point chosen as a reference. For example, the sidereal year has the fixed equinox as its reference point, which is not affected by precession or nutation. In the anomalistic year, the reference point is the perihelion.

Another type of year is the seasonal years, defined as the time elapsed between two consecutive similar seasonal points. We then have four seasonal years: spring, summer, autumn and winter. We distinguish between true and mean seasonal years, which are the ones of interest for applying them to the calendar. The calculation of mean seasonal years is done by applying (5.5).

In Table 3.5, we have included the four mean seasonal years from the year -1000 to the year 3000. We find that the length of the years is different from each other, with a difference that can exceed one minute. The other characteristic that we observe in Table 3.5 is that the mean seasonal years vary over time, with some increasing and others decreasing.

Table 4.5 shows the mean seasonal years from -1000 to 3000. We see that the average of the four years coincides very approximately with the tropical year of the date, which allows us to define the tropical year approximately as the mean value of the four seasonal years.

The Chinese calendar is governed by the winter seasonal year, i.e. the time between two winter solstices. We can see in Table 3.5 that the length of the average winter year is currently decreasing slightly by 13 seconds every millennium.

In Illustration 3.5, we find the duration of the true winter year from 2000 to 2040. We see that there is a large dispersion of the dates with respect to the average value. The difference between the extreme values exceeds twenty minutes.

Date of passage of the Sun through perihelion

Perihelion is the point in the Sun's orbit closest to the Earth's centre. Using the

Gregorian year	Spring year	Summer year	Autumn year	Winter year
-1000	365.242 094	365.241 866	365.242 724	365.242 879
0	365.242 140	365.241 728	365.242 498	365.242 885
1000	365.242 261	365.241 648	365.242 256	365.242 840
2000	365.242 374	365.241 626	365.242 017	365.242 740
3000	365.242 463	365.241 658	365.241 799	365.242 592

Table 3.5.- **AVERAGE SEASONAL YEARS**. We calculate the mean seasonal years for different periods. We find that there are differences between each seasonal year and that they also vary unequally over time. For example, in the period under consideration, the mean spring year increases, the summer year remains almost constant, and the autumn and winter years decrease. In other periods, the variations will be different. In the year 2000 the extreme differences between two seasonal years (winter and summer) were 80.7 seconds, a difference that reached 108 seconds in year 0.

Gregorian year	Spring year	Summer year	Autumn year	Winter year	Sum of differences (seconds)
-1000	- 29.8	- 44.0	30.2	43.6	0.0
0	-14.9	- 50.5	16.0	49.5	-0.02
1000	0.9	- 52.1	0.4	50.8	-0.01
2000	15.9	- 48.7	- 14.9	47.6	0.01
3000	28.9	- 40.6	- 28.4	40.1	0.0

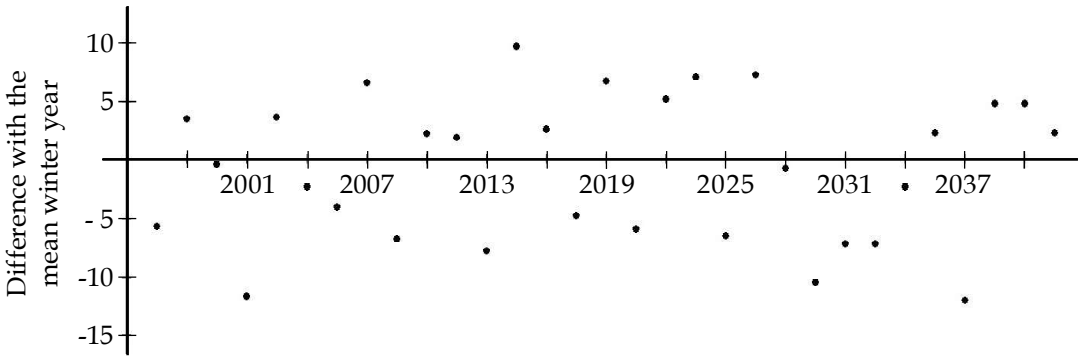
Table 4.5.- **SEASONAL YEARS AND TROPICAL YEARS**. The table shows the seasonal years' excesses in seconds over the tropical year of the date. It also shows, for various periods, the difference in seconds between each of the four seasonal years and the tropical year of the date. The last column is the sum of the four differences. The sum of the four seasonal years is approximately equal to four times the tropical year, or the tropical year is approximately the average of the four seasonal years.

VSOP87 theory, we determine the Earth-Sun distance and find the moment when the smallest distance is reached between the centres of the Sun and the Earth, which corresponds to the instant of the Sun's passage through perihelion.

In Table 5.5, we have calculated the times of passage of the mean Sun through the mean perihelion, whose longitude is (2.5). The times of passage of the true Sun through the true perihelion may differ by more than one day.

In Table 6.5, we have calculated the times of the Sun's passage through true perihelion and true aphelion, which occur between January 3 and 5 for perihelion and between July 4 and 6 for aphelion, using the time in the Beijing time zone.

From (2.5), we see that for the years in Table 6.5, the mean ecliptic longitude of



Ilustration 3.5.- **DISPERSION OF THE LENGTH OF THE TRUE WINTER SEASONAL YEAR**. The true winter seasonal year is the time between two consecutive winter solstices. The horizontal axis of the graph shows the Gregorian years. The vertical axis shows the minute difference between the true and mean winter years. The points correspond to the winter solstice date according to the Beijing time zone. We see that there is a large dispersion around the mean value. The extreme values of the years shown in the graph are separated by more than 21 minutes.

Year	Date	Hour : minute (UTC+8)
2024	2024/01/04	13:43
2025	2025/01/03	19:57
2026	2026/01/04	02:11
2027	2027/01/04	08:25
2028	2028/01/04	14:38
2029	2029/01/03	20:53
2030	2030/01/04	03:06
2031	2031/01/04	09:20
2032	2032/01/04	15:34
2033	2033/01/03	21:48
2034	2034/01/04	04:02
2035	2034/01/04	10:15

Table 5.5.- **PASSAGE OF THE MEAN SUN THROUGH MEAN PERIHELION.** The time is expressed in UTC relative to the Beijing time zone. The differences from year to year are due to the irregular intercalation rule for leap years. The average time between two consecutive passages of the Sun through the perihelion is the mean anomalistic year, which for the year 2000 is $365^{d}\,6^{h}\,13^{m}\,53^{s}$. The results have been obtained by equating the mean longitude of the Sun (1.5) with the longitude of the perihelion (2.5).

$$L_m + 360 \cdot (Y - 2000) = \bar{\omega}_m ,$$

Y is the year.

Year		Date	Hour : minute	Longitude	Distance (AU)
2024	Perihelion	2024/01/03	08:49	282-07	0.983 306 879
	Aphelion	2024/07/05	12:54	103-41	1.016 726 210
2025	Perihelion	2025/01/04	21:47	284-28	0.983 327 357
	Aphelion	2025/07/04	04:06	102-09	1.016 643 770
2026	Perihelion	2026/01/04	01:03	283-20	0.983 302 291
	Aphelion	2026/07/07	02:05	104-41	1.016 644 345
2027	Perihelion	2027/01/03	10:18	282-27	0.983 333000
	Aphelion	2027/07/05	13:26	103-00	1.016 728 914
2028	Perihelion	2028/01/05	20:16	284-41	0.983 307 923
	Aphelion	2028/07/04	06:18	102-29	1.016 680 266
2029	Perihelion	2029/01/03	02:36	282-40	0.983 291 361
	Aphelion	2029/07/06	13:00	104-26	1.016 712 173
2030	Perihelion	2030/01/03	18:57	283-07	0.983 340 082
	Aphelion	2030/07/04	20:54	102-37	1.016 722 544
2031	Perihelion	2031/01/05	05:00	284-19	0.983 266 917
	Aphelion	2031/07/06	14:16	104-2	1.016 687 750
2032	Perihelion	2032/01/03	13:07	282-22	0.983 247 864
	Aphelion	2032/07/05	19:55	104-01	1.016 751 220
2033	Perihelion	2033/01/04	19:20	284-25	0.983 298 769
	Aphelion	2033/07/04	05:11	102-15	1.016 693 552
2034	Perihelion	2034/01/04	12:27	283-52	0.983 290 292
	Aphelion	2034/07/06	03:24	104-49	1.016 670 476
2035	Perihelion	2035/01/03	08:53	282-27	0.983 325 600
	Aphelion	2034/07/06	02:29	103-36	1.016 740 613

Table 6.5.- **THE SUN'S PASSAGE THROUGH PERIHELION AND APHELIUM**. The table shows the day and time of the Sun's passage through perihelion and aphelion (closest and farthest distance from the Sun), as calculated by the simplified VSPO87 theory. The times are referred to the time zone where Beijing is located, i.e. 120º east of Greenwich and expressed on the Coordinated Universal Time scale. In the fifth column is the geometric ecliptic longitude of the Sun when it is at aphelion or perihelion. In the last column is the Earth-Sun distance expressed in astronomical units. Perihelion passage always occurs at the current time in solar month Z11 (see below), i.e. between the major solar terms Z11 and Z12. Aphelion passage of the Sun always occurs in solar month Z5, i.e. between the major solar terms Z5 and Z6. The dispersion is measured by the root mean square deviation of the perihelion longitudes, which is 55 arc minutes.

perihelion is approximately 283º 25′. We also see from Table 6.5 that there is a large dispersion in the true perihelion longitude, with a root mean square deviation of 55 arc minutes; in the period considered, dispersions of the perihelion longitude of more than one degree with respect to the mean perihelion longitude are found.

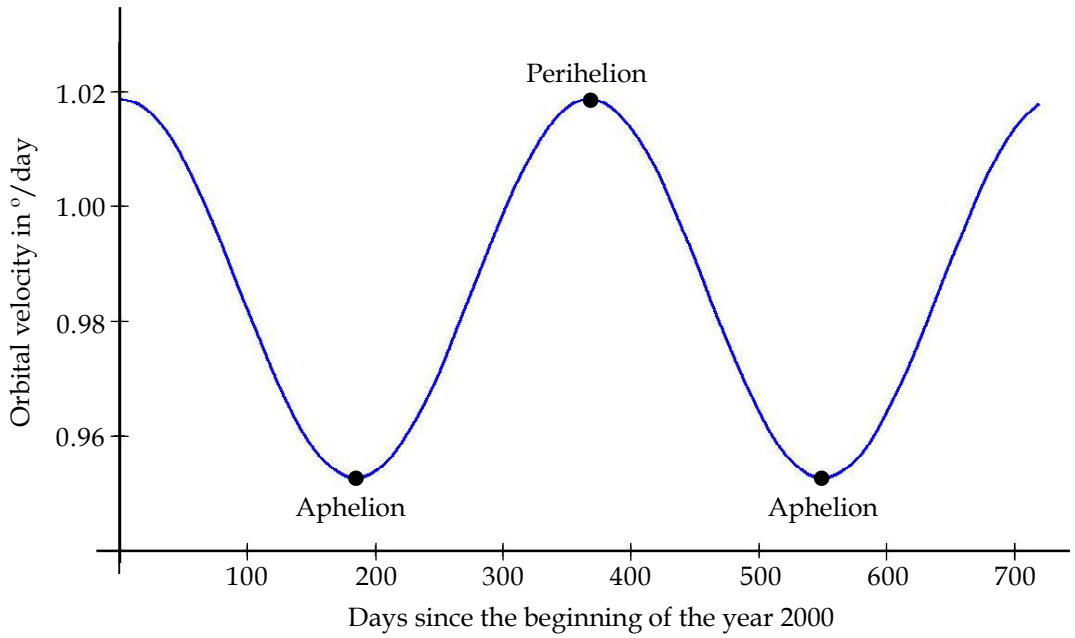
Orbital velocity of the Sun

Kepler's second law of planetary motion states that orbital velocity is variable, depending on the position in the orbit, with the velocity being maximum at the pericenter and minimum at the apocenter.

In (5.5), we express the Sun's ecliptic longitude, considering only the centre's equation and neglecting the remaining periodic terms. From (5.5), we find an estimate of the apparent orbital velocity of the Sun

$$v = \frac{dL}{dT} = \frac{dL_m}{dT} + \left[\left(2e - e^3/4\right)\cos M + 5/2\, e^2 \cos 2M + 13/4\, e^3 \cos 3M\right]\frac{dM}{dT} \qquad (6.5)$$

v is the orbital velocity. The expressions for L_m and M as functions of time are (1.5) and (3.5). Illustration 4.5 shows the variation of the Earth's mean orbital velocity in its annual motion. We see that the velocity is maximum at perihelion, where it reaches a velocity of approximately 1.02 degrees per day. In contrast, the minimum velocity occurs at aphelion, where its velocity is 0.95º/day. As we will see later, this difference in velocity has significant consequences for the Chinese calendar.



Ilustration 4.5.- **MEAN ORBITAL VELOCITY OF THE SUN**. The horizontal axis shows the days elapsed since January 1, 2000. The vertical axis shows the velocity in degrees per day. The maximum orbital velocity corresponds to the Earth passing through the perihelion and the minimum when it passes through the aphelion. The difference between the maximum and minimum velocities is slight because the Earth's orbit is not very eccentric. However, these variations are important in the Chinese calendar. The results were obtained by equation (5.5) and by $v = dL/dT$, v is the orbital velocity.

Table 7.5 shows the Sun's true orbital velocity at true perihelion from 2024 to 2035. The velocity remains practically constant.

Precession of perihelion

We have observed a periodic variation in the position of the perihelion, which has an amplitude of approximately one degree with respect to the mean position. In addition to this periodic variation, the perihelion is subject to a secular variation, which is given by (2.5). This precession of the perihelion is in a direct sense, that is, in the same sense as the proper motion of the Sun.

Illustration 5.5 shows the secular variation of the perihelion position deduced from (2.5), which is a nearly linear expression. From (2.5), we find that each century, the longitude of the perihelion increases by 1.719 degrees on average; as the orbital velocity is approximately 1^{o}/day, then in each century, the perihelion will arrive 1.7 days later. Indeed, in Illustration 6.5, we see that the Sun's passage through the perihelion is becoming later and later, being January 16th for the year 2700.

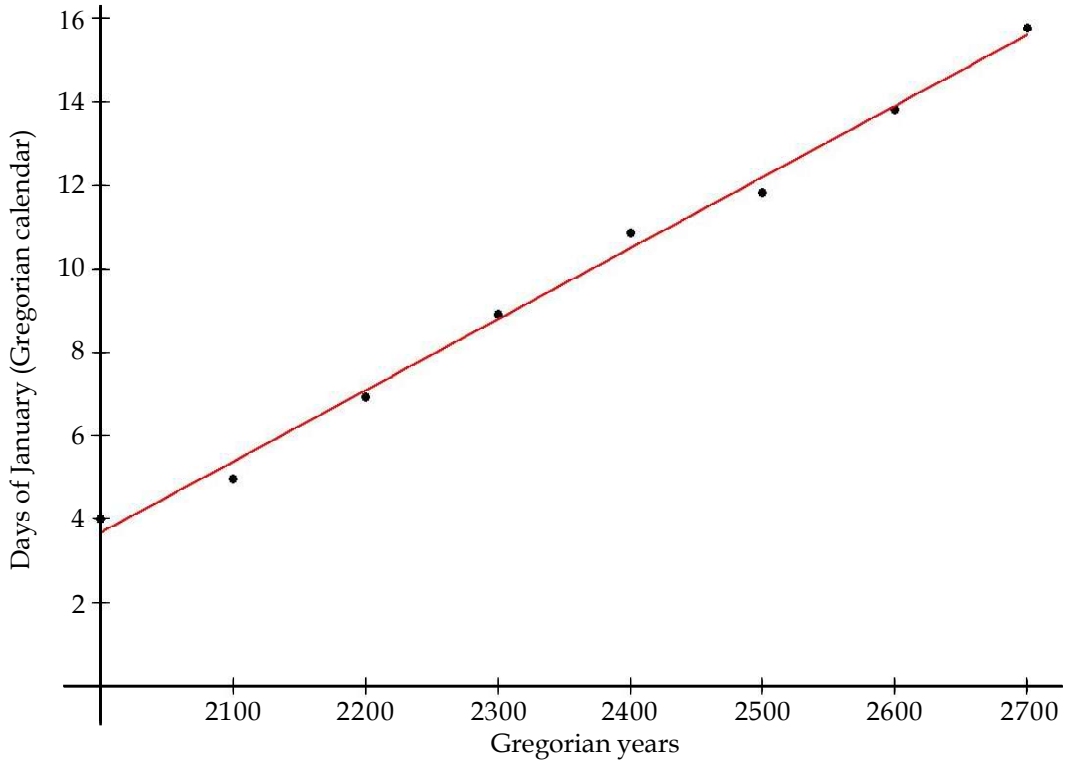
Since solar months (see below) are shorter when the Sun is in the perihelion zone, the secular precession of perihelion will have consequences for the future of the Chinese calendar.

Orbital elements of the Moon

The Moon describes an approximately elliptical orbit around the Earth, strongly perturbed mainly by the action of the Sun. The semi-major axis of the orbit has an

Date	Hour : minute	Velocity (^{o}/day)
2024/01/03	08:49	1.0192
2025/01/04	21:47	1.0194
2026/01/04	01:03	1.0188
2027/01/03	10:18	1.0195
2028/01/05	20:16	1.0192
2029/01/03	02:36	1.0189
2030/01/03	18:57	1.0187
2031/01/05	05:00	1.0190
2032/01/03	13:07	1.0192
2033/01/04	19:27	1.0196
2034/01/04	12:27	1.0189
2035/01/03	08:53	1.0193

Table 7.5.- **ORBITAL VELOCITY OF THE SUN AT PERIHELION.** The table shows the Sun's orbital velocity as it passes through the true perihelion. We observe that the velocity is practically constant.



Ilustration 5.5.- **AVERAGE DATE OF THE SUN'S PERIHELION PASSAGE.** The graph shows the precession of perihelion, which causes the Sun to pass through perihelion later and later in the Gregorian calendar. Approximately every century, the Sun passes through perihelion 1.7 days later. This shift will have consequences for the Chinese calendar. Shortly before the year 3000, the moment of passage through the perihelion will be in a different solar month (see below) than it is at present. The points correspond to the average moment of passage of the Sun through perihelion on the date indicated; the points do not coincide with the straight line of fit due to the irregular intercalation of leap years in the Gregorian calendar.

length of 384,399.07 kilometres. The lunar orbit is comparable to an ellipse with a mean eccentricity of 0.0549. However, solar perturbation causes the eccentricity to vary, taking extreme values of 0.045 and 0.065, which causes the distance between the Earth and the Moon to vary substantially with respect to the mean value.

The Moon's orbital plane is inclined to the ecliptic at an average angle of 5º 8′ 43′′, but it undergoes periodic variations; the most important of these is a semi-amplitude of 9 minutes and a period of 173 days. The direction of the Moon's motion relative to the ecliptic is the same as that of the Sun and the rest of the planets, that is, a direct motion.

The line of apses, which connects the perigee to the apogee, undergoes a rapid rotation in a direct motion, with a period of approximately 8.85 years. The line of nodes also undergoes a rapid movement in the retrograde motion, with a period of 18.61 years.

For the description of the lunar motion, we will use the theory called Parisian Lunar Ephemeris ELP of Chapront-Touzé and Chapront. The mean orbital elements of the Moon are given in polynomial form as a function of time, expressed as usual,

in Julian centuries counted from the beginning of the year 2000. The mean elements of the Moon's orbit are

$$L'_m = 218^\circ.3164591 + 481\,267^\circ.881\,342T - 0^\circ.0013268T^2$$
$$M' = 134^\circ.9634114 + 477198^\circ.8676313T + 0^\circ.0089970T^2$$
$$\Omega_m = 125^\circ.0443598 - 1934^\circ.136185T + 0^\circ.0020762T^2$$
$$\varpi'_m = L_m - M = 83^\circ.353\,0477 + 4069^\circ.013\,711T - 0^\circ.010\,3238T^2,$$

L'_m is the mean longitude, M' is the mean anomaly (measured from the mean perigee to the mean Moon position), Ω_m is the longitude of the ascending node of the lunar orbit, and ϖ'_m is the longitude of the mean perigee.

We call elongation the difference between the ecliptic longitudes of the Moon and the Sun. The mean elongation D_m is the difference between the mean ecliptic longitudes.

For some calculations that we will do later, it is sufficient to consider the first periodic terms. The lunar ecliptic longitude is approximately

$$L = L_m + 22639''.59\sin M + 769''.02\sin 2M + 36''.12\sin 3M + 4586''.50\sin(2D_m - M) + \\ +2369''.93\sin 2D_m - 666''.42\sin M' - 124''.99\sin D_m - 411''.60\sin(2L_m - 2\Omega_m) \tag{7.5}$$

terms 2, 3, and 4 are those corresponding to the center equation, 5 is the evection, 6 is the variation, 7 is the annual equation, 8 is the paralactic term, and 9 is the reduction to the ecliptic.

Synodic month and anomalistic month

The synodic month or lunation is the time elapsed between two consecutive Sun and Moon conjunctions, i.e., the period between two new Moons. We distinguish between true and mean lunation.

Because of the periodic terms of the motions of the Sun and Moon, the lunation or true synodic month is a variable time. The mean lunation is the time that must pass for the mean elongation between the Sun and the Moon D_m to increase by 360°. The mean synodic month has a secular variation with time but does not contain periodic terms.

The synodic mean motion or motion of the Moon relative to the Sun is

$$n_s = \frac{dD_m}{dT} = \frac{d}{dT}(L'_m - L_m) = 445\,267^\circ.111\,5193 - 0.003\,260\,958T \tag{8.5}$$

expressed in degrees per Julian century; in degrees per day it is

$$n_s = 12.190\,749\,118 - 8.928\cdot 10^{-8}T,$$

n_s changes very slightly over time, decreasing by 11.74 arc seconds per century each century, so during a lunation, we can understand that it remains constant; this allows us to calculate the length of the synodic month by the relationship

$$m_s = \frac{360}{n'} = 29^d.530\,588\,85 + 2^d.163\cdot 10^{-7}T \tag{9.5}$$

with units of days of 24 hours of terrestrial time. Every century, the mean synodic month increases by 0.0187 seconds, therefore we consider the synodic month a constant period.

There are other definitions of months, depending on the reference point:

anomalistic, tropical, or sidereal month. However, the one of interest in calendars is the synodic month, which takes as a reference the relative position of the Moon with the Sun and is, therefore, the period of the lunar phases.

The anomalistic lunar month is the time elapsed between two consecutive passages of the Moon through its perihelion or the time it takes to increase its anomaly (angle between the perigee and the Moon) by 360º. The anomalistic mean movement or velocity is

$$n_a = \frac{dM'}{dT} = 477\,198^{\circ}.867\,6313 + 0.017\,994T$$

in degrees per Julian century, from which we deduce that the mean anomalistic month is

$$m_a = 27^d.554\,550 - 1.039 \cdot 10^{-6}\,T. \tag{10.5}$$

Chapront's ELP-2000/82 theory

Jean Meuss developed a simplified lunar theory based on Chapront's ELP-2000/82 lunar theory. The theory comprises a series of sixty terms that calculate the Moon's ecliptic longitude, latitude, and Earth-Moon distance. This Meuss adaptation is approximately 10 arcseconds accurate for longitude and 4 for latitude, corresponding to an error of 3.5 seconds.

The terms of the series have the form

$$\begin{aligned} L' &= L'_m + \sum \frac{L'_i}{1000000} \sin\left[a_i D_m + b_i M + c_i M' + d_i \eta'_m\right] \\ \lambda' &= \sum \frac{\lambda'_i}{1000000} \sin\left[e_i D_m + f_i M + g_i M' + h_i \eta'_m\right] \\ r' &= 385000.56 + \sum \frac{r'_i}{1000} \cos\left[j_i D_m + k_i M + l_i M' + m_i \eta'_m\right] \end{aligned} \tag{11.5}$$

L'_i, λ'_i, and r'_i are numerical coefficients obtained from the Appendix II. The first two are measured in millionths of a degree, and r'_i is measured in meters. Therefore, the ecliptic latitude L' and latitude λ' are expressed in degrees, and the Earth-Moon distance is expressed in kilometres. The coefficients $a_i, b_i, ..., m_i$ are dimensionless integers obtained from the Appendix II.

Some corrections are necessary. The secular variation of the Earth's eccentricity must be considered, which appears in all the terms with the mean solar anomaly M in their argument. Thus, the summands of (11.5) that contain M (Appendix III) must be multiplied by the corrective element E, and those that contain $2M$ must be multiplied by E^2, being

$$E = \frac{e}{e_0} = 1 - 0.002\,516T - 0.000\,0074T^2$$

e is the eccentricity of the Earth and e_0 is the value of the eccentricity at time $T = 0$.

On the other hand, to the ecliptic longitude calculated by (11.5) we must add the terms

$$14''.25 \sin A_1 + 7''.06 \sin\left(L'_m - \eta'_m\right) + 1''.14 \sin A_2$$

reflecting the planetary perturbations of Venus and Jupiter and those produced by the flattening of the Earth; the coefficients are

$$A_1 = 119^\circ.75 + 131^\circ.849T$$
$$A_2 = 53^\circ.09 + 479\ 264^\circ.290T$$
$$A_3 = 313^\circ.45 + 481\ 266.484T.$$

To the ecliptic latitude of (11.5) we must add

$$-8''.05\sin L'_m + 1''.38\sin A_3 + 0''.63\sin\left(A_1 - \eta'_m\right) +$$
$$+0''.63\sin\left(A_1 + \eta'_m\right) + 0''.46\sin\left(L'_m - M'\right) - 0''.41\sin\left(L'_m + M'\right).$$

Correcting the nutation, which gives the longitude with respect to the true equinox, is necessary. The nutation by longitude is given

$$\Delta L' = -17''.20\sin\Omega_m - 1''.32\sin\left(2L_m\right)$$

quantity that must be added to the first equation of (11.5) to obtain the apparent longitude of the Moon.

Lunar period of 412 days

The lunation or interval between two consecutive conjunctions of the Sun and Moon is variable due to both stars' secular and periodic variations. The duration of the lunation depends on the position of the Moon at the beginning of the lunation and the relative position of the lines of the apses of the Sun (line that joins perihelion and aphelion).

Illustration 6.5 represents the orbit of the Moon. Suppose that the Moon is at point 1 and in conjunction with the Sun, which begins a lunation or synodic period. While the Moon describes its path, the Sun, initially in the same direction as 1, moves in the same sense as the Moon but slower. After a while, the directions of both stars with respect to the Earth coincide at point $1'$, concluding the lunation.

The lunation is the time it takes for the Moon to travel the arc $11+11'$. The Moon travels the arc 11 in an anomalistic month; the arc 11' is in the vicinity of the lunar perigee (as shown in Ilustration 6.5); therefore, the Moon will travel it in a short time. In this situation, the lunation will be shorter than average.

The reverse situation to the previous one occurs when the Moon is in conjunction at point 2. At point 2' the directions of the Sun and the Moon will coincide again. As before, the lunation will be the anomalistic month added to time that the Moon takes to travel the arc 22',

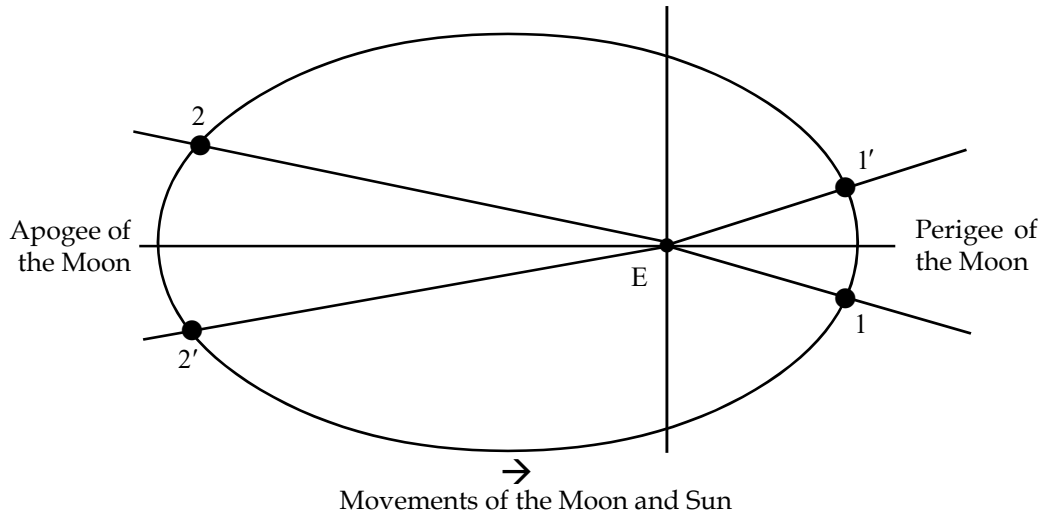
For the above, we say that the longest lunations are when the Sun is closely aligned with the apogee of the lunar orbit; if in this optimal situation there is a conjunction of Sun and Moon, we will have a maximum lunation. To know the time of this favorable relationship between the Sun and the lunar apogee is necessary to find out the relative velocity of the Sun with respect to the apogee of the Moon, which on average is the difference between the mean motions of the Sun and the perigee of the Moon

$$\dot{L}_m - \bar{\omega}'_m \approx n - \bar{\omega}'_m$$

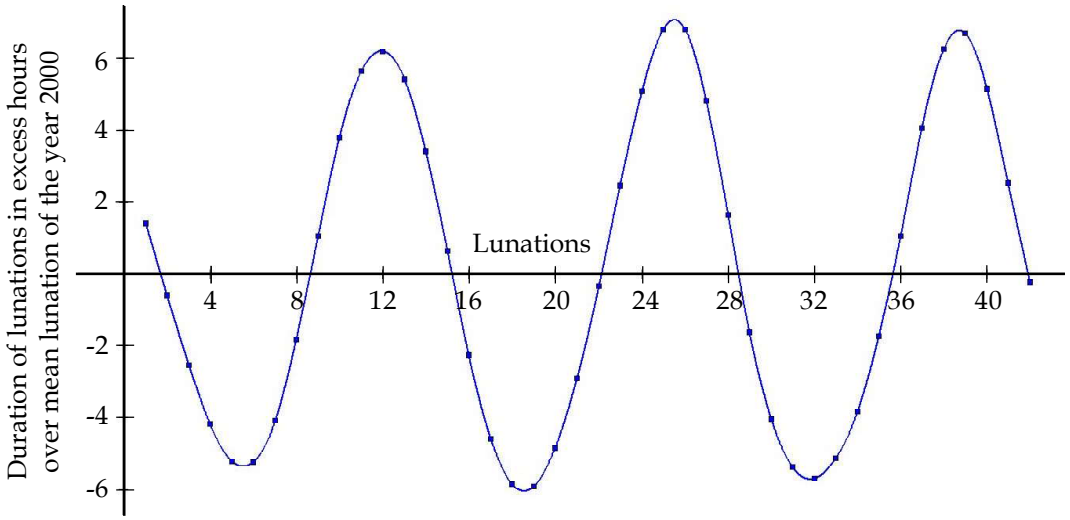
and the period of this movement or time elapsed between two crosses of the Sun by the lunar pericenter is

$$\tau_1 = \frac{360^\circ}{n - \bar{\omega}'_m}$$

which is the first period or short period of variation of the lunation. We determine



Ilustration 6.5.- **MAXIMUM AND MINIMUM OF THE LUNATION.** Orbit of the Moon around the Earth *E*. The lunation that begins with the conjunction at 1 and ends 1' is shorter than the average, because the Moon moves through the perigee where it has more velocity than average. The lunation that begins at 2 and ends at 2′ is longer than average since the Moon has a lower velocity at the apogee. If at the beginning of the lunation at 2, the Sun is at perihelion, the distance 22′ travelled by the Sun will be greater, and therefore, the Moon will take longer to travel from 2 to 2'; that is, the lunation is even longer if the Sun is at perihelion during the conjunction.



Ilustration 7.5.- **412-DAY PERIOD OF THE LUNATION.** We represent the durations of the 42 lunations after the beginning of the year 2025. The vertical axis is the hours of excess or defect of the lunation with respect to the mean lunation of the year 2000. On the horizontal axis, we number the lunations from the year 2025. We find the first maximum in lunation 12 and the next maximum in lunation 26, a period of approximately 14 lunations: the 412 days considered in the text.

the denominator of the previous expression, resulting in the year 2000 that the short period of the lunations is on average

$$\tau_1 = 411^{\mathrm{d}}.78 \approx 412^{\mathrm{d}}$$

or approximately 14 lunations, which means that there will be a lunation period every 412 days or 14 lunations, passing its durations through a maximum and a minimum. In Illustration 7.5, we represent the lunations that begin in the year 2025. We see that they have a periodic variation, with an approximate period of 14 lunations. In the time interval considered, the maximum lunation is $29^{\mathrm{d}}\,19^{\mathrm{h}}\,32^{\mathrm{m}}$, and the minimum is $29^{\mathrm{d}}\,6^{\mathrm{h}}\,49^{\mathrm{m}}$ a difference of $12^{\mathrm{h}}\,43^{\mathrm{m}}$.

The duration of the lunation is not commensurate with the period τ_1; this means that the short period of the lunation is not always 14 lunations; that is, the period will change. In Ilustration 8.5, the duration of 300 lunations or 22 periods appear, and we verify that there are periods of 13 y 14 lunations, but there are also periods of 15 and 16 lunations.

9-year period of the lunation

The position of the Sun also intervenes in the duration of the lunation. Suppose that the conjunction takes place at point 1 of Ilustration 6.5. If the Sun is at its apogee, it will go slower, and therefore the arc that moves during the lunation will be shorter; this arc will be traversed by the Moon in a shorter time. We will then have shorter than average lunations.

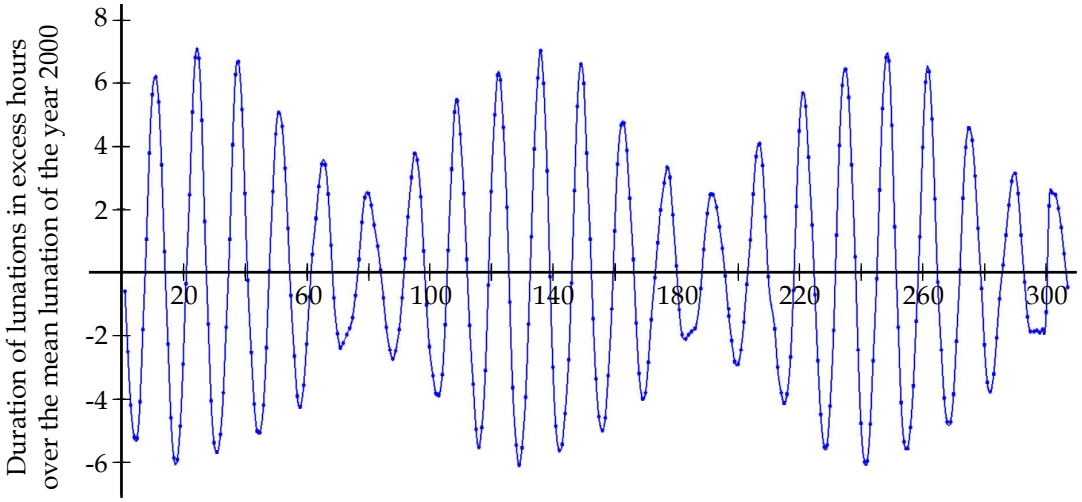
We have the opposite situation when the conjunction occurs at point 2 in Ilustration 6.5, and the Sun is in its perihelion, then the arc 22′ will be larger than average because the Sun goes faster and the Moon will take longer to travel it, having longer lunations.

Therefore, lunations will be longer when the conjunction occurs with the Moon at the apogee and the Sun at the perihelion and vice versa. Suppose conjunction occur in the previous situation, we want to know the time that has to elapse for the same situation to occur again; that is, there is a coincidence of the Moon, the Sun, the lunar apogee, and the solar perihelion.

The period we are looking for is a whole number of lunations since both the beginning and the end of the cycle are conjunctions. When that number of lunations elapses, the apogee of the Moon, the perihelion of the Sun, and the Moon (and naturally the Sun) must coincide. That is, during that number of lunations, a whole number of anomalistic months must have elapsed since the Moon returns to the same starting point with respect to the lunar orbit, and in addition, there must be a whole number of turns of the solar perihelion with respect to the apogee lunar.

The anomalistic month is the time between two passages of the Moon through the perigee; its average value is 27.555 days (10.5). Next, it is necessary to determine the time between two consecutive coincidences of the lunar apogee and the solar perihelion, for which it is necessary to know the relative velocity of the lunar perigee with respect to the solar aphelion, which is the difference of the mean movements of the pericenters of both stars $\omega_m - \omega'_m$, then the time elapsed between two crosses of the solar perihelion by the lunar apogee is

$$\tau_2 = \frac{360^{\mathrm{o}}}{\dot{\bar{\omega}}_m - \dot{\bar{\omega}}'_m} \tag{12.5}$$



Ilustration 8.5.- **9-YEAR PERIOD OF THE LUNATION.** We represent the duration of 300 lunations beginning with the year 2025. The vertical axis is the duration of the lunation in hours of excess over the mean lunation of the year 2000. The horizontal axis is the lunations beginning with the first of the year 2025. In addition to the 412-day period, we see another period of the maximum and minimum lunations, which have a duration of approximately 111 lunations or 9 years. The maximum duration of the lunation in the considered period is $29^d\ 19^h\ 46^m\ 12^s$ el día 2036/01/28, and the lunation of shorter duration is $29^d\ 6^h\ 38^m\ 39^s$ el día 2035/07/05, difference of $13^h\ 7^m\ 33^s$.

we obtain as an average value of this second period, $\tau_2 = 8.85$ Julian years (= 3,231.5 days), which is approximately 109-110 lunations.

Now we have to find an whole number of lunations x, which is also an iwhole y of anomalistic months and an whole z the period τ_2; that is, we have to find the coefficients x, y, z in the expression

$$29.5306x = 27.555y = 3\ 231.5z.$$

We look for an approximate solution since the periods considered are not congruent with each other; that is, we have to look for coefficients x, y, and z closest to whole numbers. The best solution is

$$x = 111; \quad y = 1.01436; \quad z = 118.958.$$

If we start when all the points coincide, then after the 111 lunations have elapsed, the solar perihelion is at $5^o\,(= 0.01436 \times 360)$ from the lunar apogee, and the conjunction occurs at $-15^o\,(= 0.958 \times 360)$ of the lunar apogee. Naturally, not all periods can have the exact duration of 111 lunations because the errors above would accumulate. That is, the periods of 111 lunations will be inserted with other periods that, on average, keep the conjunctions close to the lunar apogee and solar perihelion.

In Iustration 8.5, we have represented the duration of 300 lunations starting with the first of the year 2025. We observe the first period of 412 days. However, we also observe that the maximums and minimums have the second period of approximately 111 lunations, as we have calculated.

In the millennium that begins in the year 2025, the maximum lunation is $29^d\ 19^h\ 54^m\ 21^s$ and will end with the New Moon on December 19, 2150. The lunation of less duration in the same period is $29^d\ 6^h\ 35^m\ 5^s$ and will end on June 17, 2053. A

difference of 13 hours and 19 minutes.

Effect of the variation of the Earth's eccentricity on lunation

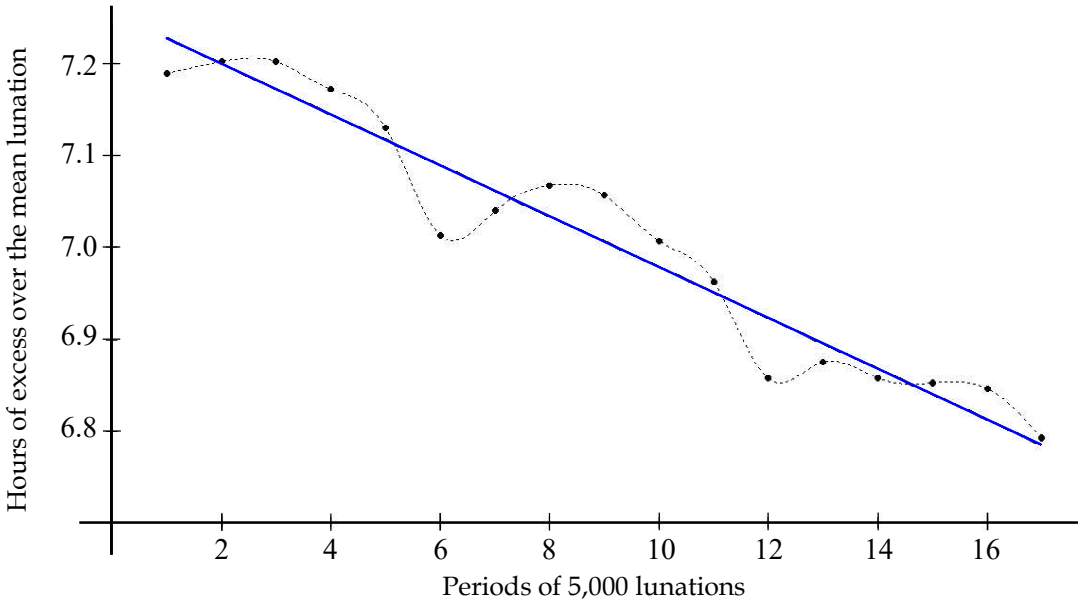
The secular decrease in the eccentricity of the Earth is another factor that intervenes in the duration of lunations. In Ilustration 9.5, we show the maximum lunation durations in periods of 5,000 lunations, starting the count in the year 2000. We observe that the values are decreasing in an approximately linear way. On the contrary, the minimum durations of lunations follow an upward trend, also linearly.

We verify that the eccentricity e of the Earth decreases secularly with time. Before, we have verified that the maximum durations occur when the Sun is in its perihelion ($M = 0$), we verify that when e decreases, the synodic movement n_s increases, that is, the duration of lunations decreases.

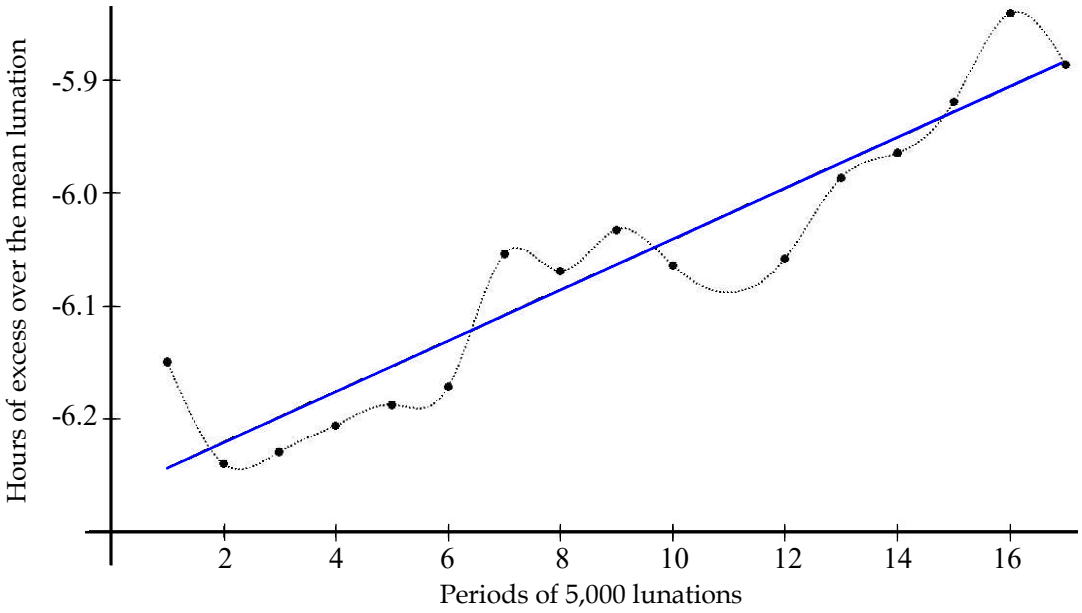
However, when the minimum lunations occur, the Sun is at its aphelion, with its average anomaly close to 180°; as the eccentricity decreases, the same occurs with n_s at the time of the minimums, which causes the lunation to increase when the Sun is in the aphelion.

184 year period of the lunation

As we have exposed, the variation of velocities of the Sun and the Moon as they pass through the pericenter and apocenter produces a periodic variation in the



Ilustration 9.5.- **DECREASE IN THE DURATION OF MAXIMUM LUNATION.** Each point corresponds to the maximum lunation in a period of 5,000 lunations, beginning in the year 2000 and ending after 85,000 lunations. On the vertical axis are the hours of the excess over the mean lunation of the year 2000. We number the horizontal axis according to the groups of 5,000 lunations. The solid line is the curve fitting. We verify that there is a slight decrease in lunation for the decrease in the eccentricity of the Earth. Knowing the slope of the line fitting, we find an average decrease in the maximum lunation of 1.6 minutes every 5,000 lunations.



Ilustration 10.5.- **INCREASE IN THE DURATIONS OF MINIMUM LUNATION.** Each point corresponds to the minimum lunation in a period of 5,000 lunations, beginning in the year 2000 and ending after 85,000 lunations. On the vertical axis is the excess hours over the mean lunation of the year 2000. We number the horizontal axis in groups of 5,000 lunations. The solid line is the curve fitting. We verify that there is a slow increase in lunation for the decrease in the eccentricity of the Earth. By the slope of the line fitting, we find an average increase of 1.88 minutes in the minimum lunation every 5,000 lunations.

duration of the lunation.

In the same way, the variation in the velocity of the Moon as a function of its position with respect to the nodes also gives rise to another periodicity, which, as we will see, has a much longer period than the other two periods previously examined. The most favorable situation for maximum lunations is when the conjunction coincides with the Moon at perigee, the Sun at its aphelion, and the Moon at one of its nodes.

The mean velocity of the solar perihelion with respect to the ascending node of the Moon is

$$\dot{\bar{\omega}}_m - \dot{\Omega}_m$$

Ω_m is the mean velocity of the ascending node with respect to the equinox, and $\bar{\omega}_m$ is the mean velocity of the solar perigee also with respect to the equinox.

In the time interval τ_2, calculated in (12.5), the lunar perigee and the solar aphelion meet again. In this interval, the lunar perigee moves the angle $\Omega_m \tau_2$ with respect to the equinox, and the solar apogee moves an angle $\bar{\omega}_m \tau_2$ also with respect to the equinox. In this same time interval τ_2, the ascending node of the Moon moves

$$\left(\dot{\bar{\omega}}_m - \dot{\Omega}_m\right)\tau_2$$

in relation to solar aphelion. Then

$$180 - \left(\dot{\bar{\omega}}_m - \dot{\Omega}_m\right)\tau_2$$

this angle is missing to reach 180º between the solar perigee and one of the nodes, where the favorable conditions would be given for the lunation to reach the maximum value. As cycles of years τ_2 go by, the angles (13.5) will accumulate until they reach 180º, thus completing the lunation cycle that we are considering and whose period is given by a rule of three

$$\tau_3 = \frac{180\,\tau_2}{180 - \left(\dot{\bar{\omega}}_m - \dot{\Omega}_m\right)\tau_2} \tag{13.5}$$

taking the numerical value of ω_m and Ω_m and the value of τ_2, we find that this third cycle has a period of approximately 184 years. Note that in the calculation, we have considered that the lunar node has a retrograde sense, and therefore its velocity is negative. This cycle modulates the relative maximums of the 9-year period.

Full Moon

The full Moon is when the apparent longitudes of the Sun and the Moon differ by 180º. The time between the moment of the new Moon and the full Moon varies. In Illustration 11.5, the days between the new Moon and the full Moon are shown for the period between the years 2025-2029, and we can see that the maximum separation is 15.6 days and the minimum 14.1 days, or a difference of 1.5 days.

In Illustration 11.5, we find that the variation between the new and full Moons is variable, with a period of 14 months, or the period of 412 days that we have previously described, and which is caused by the variable distance between the Moon and the Earth.

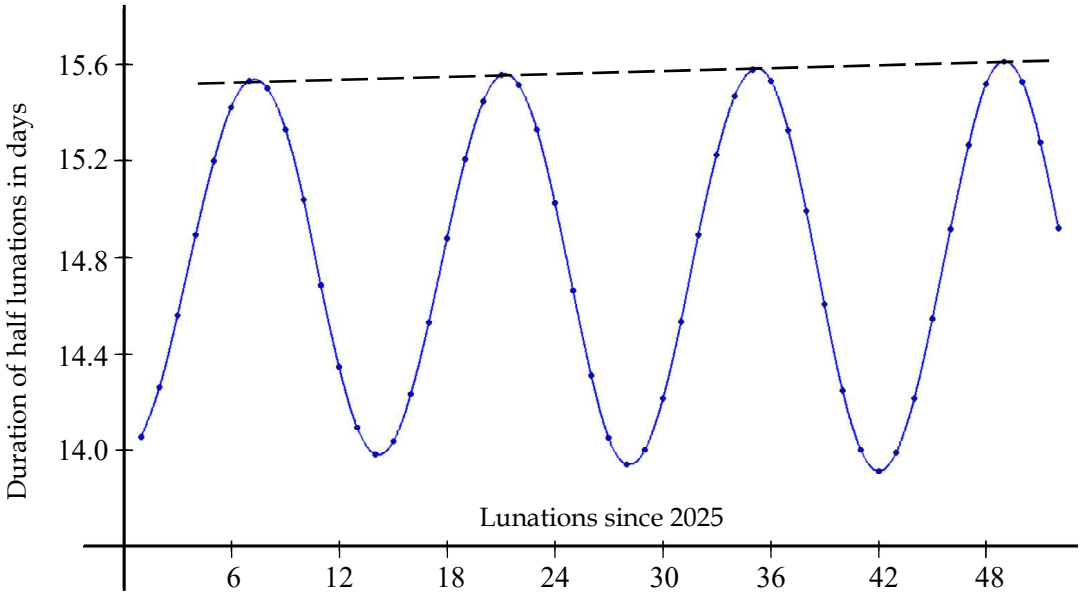


Illustration 11.5.- **SEPARATION BETWEEN THE NEW MOON AND THE FULL MOON.** We plot the duration between the new and full Moon moments from 2025 onwards, i.e. half the duration of a lunation. The latest full Moon occurs 15.5 days after the new Moon and, at the earliest, 14 days after. We confirm a periodic variation, with an approximate period of 14 lunations, i.e. the period of 412 days already described. The dashed straight line shows that the previous periods are modulated by the period of 9 years already described.

Age full Moon	Frequency	
	Number	%
14	13	0.35
15	1 399	37.70
16	1 742	46.94
17	557	15.01

Table 8.5.- **FREQUENCY OF THE AGE OF THE FULL MOON**. The full Moon can occur on the 14th, 15th, 16th and 17th days of the lunar month, where we understand that the day of the new Moon is the 1st. The table shows the frequencies in which the four possibilities can occur. The most probable is that the full Moon occurs on the 16th, and the least probable is that it occurs on the 14th. The calculations have been made for the Beijing meridian time zone. For other localities, we would obtain slightly different results.

New Moon	Full Moon
2037/06/14	2037/06/27
2049/10/27	2049/11/09
2054/05/08	2054/05/21
2057/09/29	2057/10/12
2099/06/19	2099/07/02
2165/12/04	2165/12/17
2242/07/29	2242/08/11
2259/06/22	2259/07/05
2276/05/15	2276/05/28
2287/09/09	2287/09/22
2292/03/20	2292/04/02
2295/08/12	2295/08/25
2312/07/06	2312/07/19

Table 9.5.- **FULL MOONS 14 DAYS OLD**. We show the full Moon which occurs on the 14th day of the lunar month from 2025 to 2324 in the Beijing time zone.

The dashed straight line shows a gradual increase in the maximum intervals between new and full Moons, which corresponds to another period of oscillation corresponding to the 9-year period already analyzed.

The calendar days corresponding to the new or full Moons are the days civils when these phenomena occur, starting at 0 hours of that day, regardless of the time of day when both phases of the Moon occur. We find that the day of the lunar month of the full Moon can be the 14th, 15th, 16th or 17th, with day 1 being the day of the

Duration months	Full Moon Age Frequencies			
	14	15	16	17
29	1.14 %	51.13 %	47.73 %	0 %
30	0 %	22.22 %	46.67 %	31.11 %

Table 10.5.- **FULL MOON AGE FREQUENCY.** The table shows the results found between 2025 and 2040 for the frequency of full Moons with ages 14, 15, 16 and 17 days, assuming that day 1 is the day of the new Moon. We find that a full Moon has an age of 17 days only when the lunar month of the calendar has 30 days. Full Moons with ages 15 and 16 days can occur in months with 29 and 30 days. A full Moon on day 14 can only occur in a lunar month with 29 days. The calculations refer to the Beijing time zone.

new Moon.

Table 9.5 lists the days of the full Moons in the period 2025-2324, both years included, calculated for the local time of the Beijing meridian; the results may be slightly different if another local time is used. The 16th day of the lunar month is the most probable date of the full Moon, and the 15th is also probable. The table shows that the full Moon on the 14th day of the lunar month is highly improbable. Table 8.5 shows the occasions when the full Moon has 14 days in 2025-2324.

The frequency of the Moon's age when it is full depends on the length of the month. In Table 10.5, we show the percentages calculated over a 15-year period. We find that the age of 17 days can only occur when the calendar lunar month has 30 days. We also find that the age of the full Moon is more likely to be 15 days in months with 29 days than in months with 30 days. And age 16 days occurs almost equally frequently in months of 29 and 30 days.

Duration of lunar months of the calendar

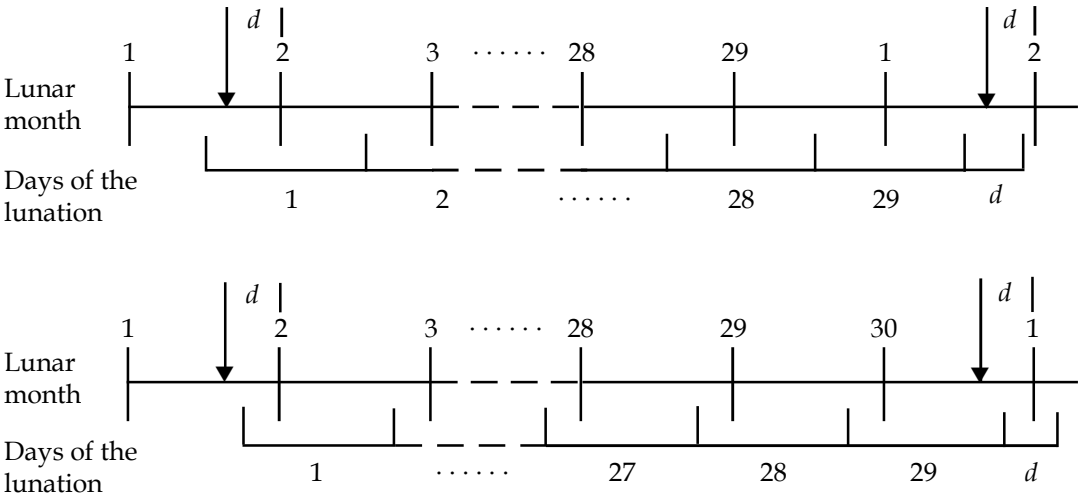
We find that the average length of the synodic month or lunation is approximately 29.53 days and that the maximum and minimum values at the current time are 29.82 and 29.28 days; that is, 29 days and a fraction of a day.

As we will see later, the lunar month of the Chinese calendar begins at 0 hours on the day of the new Moon, according to the Beijing time zone, regardless of the time of day when the new Moon occurs. This definition allows us to speak of a lunar calendar month, which only has 29 or 30 days.

The succession of 29- and 30-day months has no definite pattern. Since the average length of the lunation is approximately 29.53 days, the frequency of 29-day months will be 47% and the frequency of 30-day months 53%.

For calendar purposes, it is interesting to know how often several consecutive months with the same length can occur; we will discuss this issue later (Chapter 14). It is common to have three consecutive months of 30 days each, and it is also possible, although less likely, to have three consecutive months of 29 days each. It is also possible to have four consecutive months of 30 days each, but it is impossible to have four consecutive months of 29 days each.

The lunar month can have 29 or 30 days regardless of the astronomical lunation length. However, the frequency of this occurrence is related to the length of the



Ilustration 12.5.- **THE TWO POSSIBLE LENGTHS OF THE LUNAR MONTHS**. The diagram schematically explains the two possible lengths of lunar months. We use the example of an astronomical lunar month with a duration of 29 and *d* hours. The possible lengths of the corresponding lunar calendar month can be 29 and 30 days, depending on the time of day when the new Moon occurs. The lunar calendar month begins at 0 hours on the day when the corresponding new Moon occurs.
The drawing shows a vertical arrow marking the boundary, located *d* hours from the beginning of the second day of the lunar month. In the first drawing, we find the case where the new Moon arrives before the mark, that is, from the moment of the new Moon to the beginning of the next day, there are more than *d* hours. The end of the lunation, which lasts 29 days and *d* hours, occurs on the day after the 29th of the lunar month, so this day corresponds to the 1st of the following lunar month since the new Moon occurs in that month.
In the second drawing, we show the case in which the new Moon occurs less than *d* hours before the second day of the lunar month. The lunation ends the day 30th of the lunar month. This is the last day of the month, and the following one, the one in which the new Moon occcurs, is the first day of the new month.
Since all lunations have a duration of 29 days and a few hours, any lunation may correspond to lunar months of 29 and 30 days. Naturally, the longer the lunation, the more likely the lunar month will have 30 days and vice versa.

lunation.

A lunar calendar month has either 29 or 30 days, depending on the time of day when the new Moon occurs. Let *d* be the excess hours over 29 days of the lunation under consideration. If the new Moon arrives before $12 - d$ hours into the day, the lunar month will have 29 days (Illustration 12.5). But if the new Moon arrives after $12 - d$ hours into the day, then the lunar month will have 30 days.

6.- Computational lunisolar calendars

Computational lunisolar cycle

A computational lunisolar cycle consists of A years, which approximately fits a number of lunations L and has a duration of d days; these three quantities are whole numbers. The lunisolar cycle has common years or years with 12 lunar months and abundant years or embolismic with 13 months. The months can only have 29 or 30 days, which occur alternately whenever possible. Hence, the months of the cycle correspond to lunations, i.e., the months are lunar and begin approximately with the new Moon. In a common lunar year, there are 6 months of 29 days and another 6 of 30 days, a total of 354 days. In an embolismic year, there is also an extra month that always has 30 days, so the abundant year has a duration of 384 days *.

In summary, to develop a lunisolar cycle of A years and L lunations, we need:

- Days of the cycle (or the duration of the solar year or the lunation).
- Criterion for intercalation of the embolismic years.
- Criterion for distribution of the months of 29 and 30 days.
- Date of origin of the cycle.

Calculation of the lunations of a cycle

Knowing the number of years A of the cycle, we can determine the number of lunations it contains. Assuming year length of 365.25 days and lunation of 29.53 days on average, then

$$L \approx \operatorname{cint}\left(\frac{365.25A}{29.53}\right),$$

cintis the rounding function. For example, in an 8-year cycle, called octateris, there must be 99 lunations.

Calculation of the number of embolismic years

If y_a is the astronomical year and y_n is the length of the common year (i.e., non-embolismic) of the lunisolar cycle, the number of embolismic months E that the

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* As we will see later, in the lunisolar cycles there are several types of lunar years according to their duration. For example, in the Hebrew calendar, the duration of normal years can be 353, 354, and 355 days, and abundant years are 383, 384, and 385 days. In the Christian calendar there can also be years of 386 days.

cycle has is

$$E = \text{cint}\left[\left(y_a - y_n\right) A/D\right]$$

and D is the days of the embolismic month, which in computational lunisolar cycles always lasts 30 days. y_a only has to be an appro-ximation to the astronomical year, so it is sufficient to take it equal to the duration of the Gregorian year 365.2425 days.

For example, if the lunisolar cycle is 8 years or octaeteris, then

$$E = \text{cint}\left[\left(y_a - y_n\right) A/D\right] = \text{cint}\left[\left(365.2425 - 354\right) \cdot 8/30\right] = 3$$

therefore the 8-year lunisolar cycle has 3 embolismisc years of 13 months.

Calculation of the number of embolismic months

Each common year has 12 months, so the embolismic months in a cycle where there are L lunations and A years are

$$E = L - 12A, \tag{1.6}$$

this coincides with the number of embolism years, as each embolism year can have a maximum of one extra month.

Calculating the number of months with 29 and 30 days

To find the number of months of 29 and 30 days, it is necessary to know the total number of days in the cycle, which can be determined by the average length of the year in the cycle of A years, by knowledge of the mean lunation, or because the days of the cycle are previously known. Let d be the number of days in the cycle, L the number of lunations, then

$$m_{29} + m_{30} = L$$
$$29m_{29} + 30m_{30} = d,$$

m_{29} and m_{30} are the months of 29 and 30 days of the cycle. From the above equations we find

$$\begin{aligned} m_{30} &= d - 29L \\ m_{29} &= 30L - d. \end{aligned} \tag{2.6}$$

For example, since in the octaeteris the average length of the year is 365.25 days, then the total number of days in the cycle is 2,922, therefore months of 30 and 29 days are

$$m_{30} = 51; \quad m_{29} = 48.$$

Calculation of the average length of the year in the cycle and the average length of the lunation

The average length of the year in the A-year cycle is

$$y_m = \frac{d}{A}$$

and the average length of the lunation is

$$l_c = \frac{d}{L}.$$

The mean values of the lunation and the year are not whole numbers, but the lengths

of calendar months and years are whole numbers of days. Therefore, in addition to the two types of months, 29 and 30 days, there will be solar and lunar years of different lengths, both with integers.

For example, for octaeteris, the length of the mean solar year of the cycle is 365.25 days, and the mean lunation is

$$l_c = \frac{365.25A}{L} = \frac{d}{L} = \frac{2922}{99} = 29^{\mathrm{d}}\,12^{\mathrm{h}}\,21^{\mathrm{m}}49^{\mathrm{s}}.$$

Calculation of 29 and 30 day months not embolismic

Normal months or not embolismic can have 29 or 30 days. Since embolismic months always have 30 days, the number of normal months is

$$\begin{aligned} M_{30} &= m_{30} - E = d + 12A - 30L \\ M_{29} &= L - M_{30} - E = 30L - d = m_{29}. \end{aligned} \tag{3.6}$$

For example, for octaeteris, we find that there are 48 normal months of 29 days and another 48 of 30 days, for which we must add 3 embolismic months of 30 days.

Excesses of months of 30 days

The excess of common months (non embolismics) of 30 days over those of 29 days is

$$\Delta = M_{30} - M_{29} = 2d + 12A - 60L \tag{4.6}$$

which can be positive, negative or null.

The excess of total months of 30 days is

$$\Delta' = m_{30} - m_{29} = \Delta + E = 2d - 59L.$$

We can express Δ and Δ' in terms of the values of the lunation and the calendar year

$$\Delta = \left[\left(2l_c - 60\right)\frac{y_m}{l_c} + 12 \right] A = 2d + 12A - 60L$$

$$\Delta' = \left(2l_c - 59\right)\frac{y_m}{l_c} A = 2d - 59L.$$

For the octaetris $\Delta = 0$ and $\Delta' = 3$.

Moon leap

All embolismic months have 30 days. However, it may happen that there are not enough days in the cycle for all embolismic months to have 30 days, which occurs if Δ is negative. Then, subtract the necessary days from some common months of 30 days, which would then have 29 days, and enough days left for all embolic months to have 30 days.

By doing this operation, we assume that common months from which we remove a day still have 30 days, but we skip a day, making them 29 days. This operation is a Moon leap because we «skip» a day in the lunar month.

The number of lunar leaps that a lunisolar cycle has is

$$S = E + 6A - m_{30} = L - 12A + 6A - d + 29L = -d + 30L - 6A = -\Delta/2 \tag{5.6}$$

S only makes sense when it is zero or positive. There may be no lunar leaps in a cycle, as occurs in octaeteris, where $S = 0$.

Intercalation rule for embolismic years

Now, we find the years of the cycle that must be embolismic. We assume that the embolisms are distributed as regularly as possible in the cycle. If the number of years of cycle A were a multiple of the number of E embolisms, then the years with positions that are multiples of the A/E ratio would be 13-month years. Otherwise, we assume that the years whose position in the cycle is equal to or an integer immediately greater than $k\,A/E$ will be abundant; k is a whole number that numbers the embolismic years. The above condition is equivalent to

$$Y_k = \mathrm{int}\left(k\frac{A}{E} + 1 \right) - \mathrm{int}\left[\frac{E}{kA}\mathrm{int}\left(k\frac{A}{E} \right) \right] \tag{6.6}$$

Y_k is the position in the cycle occupied by the embolism year k. Note that the last summand is equal to 1 only if the ratio $k\,A/E$ is an integer.

For example, in the octaeteris the three embolismic years occupy positions 3, 6 and 8, which correspond to the values $k = 1,2,3$.

Embolismic years elapsed

Y_k will be an embolismic year if it meets the inequalities

$$k\frac{A}{E} + 1 > Y_k \geq k\frac{A}{E},$$

multiplying by E and dividing by A we find

$$k + \frac{E}{A} > Y_k\frac{E}{A} \geq k$$

and since Y_k is an integer

$$k = \mathrm{int}\left(Y_k\frac{E}{A} \right), \tag{7.6}$$

formula that gives us the embolismic years elapsed up to year Y_k (including this year).

Other rules for intercalation of embolisms

The order derived from (6.6) is called the natural order, but we can start counting the years not from 1 of the natural order but from any other year. The result will be that the embolism years will occupy different positions. The number of possible criteria for intercalating the embolisms is equal to the number of years in the cycle.

Let b be the number of years that have elapsed since the first of the natural order until the beginning of the year that we now use as the beginning of the displaced cycle, and let a be the number of embolisms that have elapsed since the beginning of the natural order and the year located in position b (not including this year). By (7.6)

$$\alpha = \mathrm{int}\left(\beta\frac{E}{A} \right).$$

To obtain the positions of the embolismic years in the shifted order, it is necessary to make the substitutions

$$Y_k \;\rightarrow\; Y_k + \beta; \qquad k = k + \alpha,$$

then by (6.6)

$$Y_k = \text{int}\left[\left(k+\alpha\right)\frac{A}{E}+1\right] - \text{int}\left\{\frac{E}{\left(k+\alpha\right)A}\text{int}\left[\left(k+\alpha\right)\frac{A}{E}\right]\right\} - \beta. \qquad (8.6)$$

Applying (8.6) to the octaeteris we find that the possible rules of intercalation of the embolisms are

3, 6, 8 / 2, 5, 7 / 1, 4, 6 / 3, 5, 8 / 2, 4, 7 / 1, 3, 6 / 2, 5, 8 / 1, 4, 7 .

Intercalation of months of 29 and 30 days

The distribution of 29 and 30 day months is arbitrary; however, since the average lunation length is approximately 29 and a half days, alternation between 29 and 30 day months is preferred whenever possible.

In order to distribute the months as evenly as possible, we assume that, in principle, all months of the cycle have 30 days, and we determine those months in which it is necessary to remove a day and thus evenly distribute the months of 29 days. We number the days in groups of 30 from the beginning to the end of the cycle. *k* is the numbering of the embolisms, from 1 to *E*, and *n* is the number of the month that remains with 29 days counting from the beginning of the cycle, that is, from 1 to *L*. Then the months of 29 days will be those located in the positions

$$n = \text{int}\left(k\frac{d}{30m_{29}}+1\right) - \text{int}\left[\frac{30m_{29}}{kd}\text{int}\left(k\frac{d}{30m_{29}}\right)\right] \qquad (9.6)$$

the last term is 1 if $d/\left(30m_{29}\right)$ is an integer and 0 otherwise. It is necessary to know which years are embolismic or 13 months long to determine the length of lunar years.

For example, by applying (9.6) to the eight-year cycle, months of 29 days will be placed in the positions 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33, 35, 37, 39, 41, 43, 45, 47, 49, 51, 53, 55, 57, 59, 61, 63, 65, 67, 69, 72, 74, 76, 78, 80, 82, 84, 86, 88, 90, 92, 94, 96 and 98. If we assume that the embolismic years are in positions 3, 6 and 8, we deduce the sequence and duration of the years of the cycle

355 / 354 / 383 / 354 / 354 / 384 / 354 / 384 ,

and if the intercalation rule were 2, 5 and 7 then the sequence of lunar years would be

355 / 383 / 354 / 354 / 384 / 354 / 384 / 354 .

The sequence would also change if we adopted a shifted order in the alternation of months of 29 and 30 days.

The above sequence of months derived from (9.6) is called the natural order. When a new cycle begins, we repeat the previous order. Any other orders displaced from the previous one will also be a regular distribution for the months of 29 and 30 days.

In lunisolar calendars, there are six types of lunar years based on their length: defective (353 and 383 days), regular (354 and 384 days), and abundant (355 and 385 days). These years consist of 29- and 30-day months as shown in Table 1.6. It is assumed that the distribution of 29- and 30-day months is regular.

Metonic cycle

The Athenian astronomer Meton (5th century BC) devised a lunisolar cycle of 19 years and 235 lunations, with 6,940 days. Using the algorithms deduced above,

Days	Months of 30 days	Months of 29 days	Excess Δ'
353	5	7	-2
354	6	6	0
355	7	5	+2
383	6	7	-1
384	7	6	+1
385	8	5	+3

Table 1.6.- **LENGTH OF LUNAR YEARS.** Types of lunar years according to their length and assuming a regular distribution of months of 29 and 30 days. We show the number of months of 30 and 29 days that the years have. The last column shows the excess of months of 30 days in the year.

we find

$$A = 19; \quad L = 235; \quad d = 6\,940$$
$$E = 7; \quad m_{30} = 125; \quad m_{29} = 110$$
$$y_m = 365^{\mathrm{d}}.263\,1579; \quad l_c = 29^{\mathrm{d}}.53191489$$
$$M_{30} = 118; \quad M_{29} = 110$$
$$\Delta = 8; \quad \Delta' = 15; \quad S = -4\,.$$

To determine the sequence of complete months (30 days) and defective months (29 days), we assume that all months are 30 days long, and every 63 days (approximately 6,940/110), a day is subtracted from the month in which it falls, giving rise to 110 months of 29 days. Note that after the last elimination of a day, which corresponds to 110, there are 10 days left to begin a new Meton cycle, from which the same order of intercalation of the months is repeated.

By grouping the months according to the previous criterion, we find in the Meton cycle five types of lunar years according to their duration: 8 years of 354 days, 4 of 355, 1 of 383, 5 of 384 and 1 of 385, with the following sequence

355 / 384 / 354 /354 / 384 / 354 / 355 / 383 / 355 / 384 /

/ 354 / 354 / 384 / 354 / 355 / 354 / 384 / 354 / 385 .

From the preserved ancient texts, we know that the Greek Meton cycle began on June 27, 432 BC (proleptic Julian calendar), coinciding with the summer solstice .

The positions of the abundant years are determined by (8.6), which found 19 intercalation criteria based on the value of β (Table 2.6). Meton used the criterion of the second line, that is, the one corresponding to the displacement of 1 year with respect to the natural order of intercalation.

We find that the maximum dispersion is 28.26 days, which is the increase in the separation between the same date of the lunisolar cycle and and the date of the same season of each year. The average dispersion is 14.02 days, and the mean square deviation is 8.66 days. In Illustration 1.6, we show the graph with the dispersion of the seasons as a function of the year of the cycle.

Embolismic years							
3	6	9	11	14	17	19	$\beta = 0;\ \alpha = 0$
2	5	8	10	13	16	18	$\beta = 1;\ \alpha = 0$
1	4	7	9	12	15	17	$\beta = 2;\ \alpha = 0$
3	6	8	11	14	16	19	$\beta = 3;\ \alpha = 1$
2	5	7	10	13	15	18	$\beta = 4;\ \alpha = 1$
1	4	6	9	12	14	17	$\beta = 5;\ \alpha = 1$
3	5	8	11	13	16	19	$\beta = 6;\ \alpha = 2$
2	4	7	10	12	15	18	$\beta = 7;\ \alpha = 2$
1	3	6	9	11	14	17	$\beta = 8;\ \alpha = 2$
2	5	8	10	13	16	19	$\beta = 9;\ \alpha = 3$
1	4	7	9	12	15	18	$\beta = 10;\ \alpha = 3$
3	6	8	11	14	17	19	$\beta = 11;\ \alpha = 4$
2	5	7	10	13	16	18	$\beta = 12;\ \alpha = 4$
1	4	6	9	12	15	17	$\beta = 13;\ \alpha = 4$
3	5	8	11	14	16	19	$\beta = 14;\ \alpha = 5$
2	4	7	10	13	15	18	$\beta = 15;\ \alpha = 5$
1	3	6	9	12	14	17	$\beta = 16;\ \alpha = 5$
2	5	8	11	13	16	19	$\beta = 17;\ \alpha = 6$
1	4	7	10	12	15	18	$\beta = 18;\ \alpha = 6$

Table 2.6.- **INTERCALATION CRITERIA IN THE METON CYCLE.** The 19 possible criteria for the intercalation of Meton's cycle. Each line shows the embolismic years of Meton's cycle. The first line is the natural order, and the remaining lines correspond to the displaced order. The criterion used by Meton was that of the second line, corresponding to $\beta = 1$.

Julian Meton cycle

The Meton cycle is characterized by having a duration of 19 years with 235 lunations. But as we have already indicated, we need to know the days of the cycle, which we can determine by knowing the length of the year. If we adapt the Meton cycle to the Julian year of 365.25 days then the duration of the cycle will be $d = 19 \times 365.25 = 6,030.75$ days, which cannot be because the cycle must have an integer number of days.

The problem is that in 19 years, there can be four or five leap days. So, if we adjust the Meton cycle to the Julian calendar, we find two types of cycles: those with five leap days and, therefore, a duration of 6 940 days and those with four leap days and, therefore, 6,939 days.

The frequency with which the above two Meton cycles occur is three successive cycles of 6,940 days, then one cycle of 6,939 days, and so on. In reality, we find a longer cycle, of 76 years and 27,759 days, that is, the cycle of Callippus.

The 6,939-day cycle has the following characteristics:

$$A = 19;\quad L = 235;\quad d = 6\ 939$$

$$E = 7;\quad m_{30} = 124;\quad m_{29} = 111$$

$$y_m = 365^{\mathrm{d}}.210\ 526;\quad l_c = 29^{\mathrm{d}}.527\ 659\ 57$$

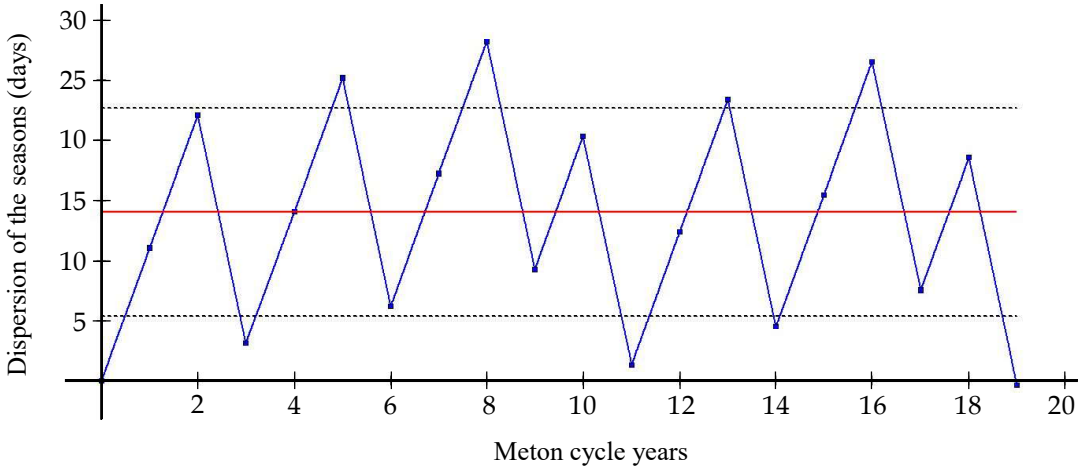
$$M_{30} = 117;\quad M_{29} = 111$$

$$\Delta = 6;\quad \Delta' = 13;\quad S = -3\,.$$

2000	2001	2002	2003	2004	2005	2006	2007	2008	2009
01/06	01/24	01/13	02/01	01/21	01/10	01/29	01/19	02/07	01/26
02/05	02/23	02/12	03/03	02/20	02/08	02/28	02/17	03/07	02/25
03/06	03/25	**03/14**	04/01	03/20	**03/10**	03/29	**03/19**	04/06	03/26
04/04	04/23	04/12	05/01	04/19	04/08	04/27	04/17	05/05	04/25
05/04	05/23	05/12	05/31	05/19	05/08	05/27	05/16	06/03	05/23
06/02	06/21	06/10	06/29	06/17	06/06	06/25	06/15	07/03	06/22
07/01	07/20	07/09	07/29	07/17	07/06	07/15	07/12	08/01	07/22
07/31	08/19	08/08	08/27	08/16	08/05	08/23	08/12	08/30	08/20
08/29	09/17	09/07	09/26	09/14	09/03	09/22	09/11	09/29	09/18
09/27	10/16	10/06	10/25	10/10	10/03	10/22	10/11	10/28	10/18
10/27	11/15	11/04	01/23	11/12	11/02	11/20	11/09	11/27	11/16
11/25	12/14	12/04	12/23	12/12	12/01	12/20	12/09	12/27	12/16
12/25		01/02			12/31		01/08		

2010	2011	2012	2013	2014	2015	2016	2017	2018	
01/15	02/03	01/23	01/11	01/30	01/20	01/10	01/28	01/24	
02/14	03/04	02/22	02/10	03/01	02/18	02/08	02/26	02/23	
03/15	04/03	03/22	**03/11**	03/30	03/20	**03/09**	03/28	03/25	
04/14	05/03	04/21	04/10	04/29	04/18	04/07	04/26	04/23	
05/14	06/01	05/20	05/10	05/28	05/18	05/06	05/25	05/23	
06/12	07/01	06/19	06/08	06/27	06/16	06/05	06/24	06/21	
07/11	07/30	07/19	07/08	07/26	07/16	07/04	07/23	07/20	
08/10	08/29	08/17	08/06	08/25	08/14	08/02	08/21	08/19	
09/08	09/27	09/16	09/05	09/24	09/13	09/01	09/20	09/17	
10/07	10/26	10/15	10/05	10/23	10/13	10/01	10/19	10/16	
11/06	11/15	11/13	11/03	11/21	11/12	10/30	11/18	11/15	
12/05	12/24	12/13	12/03	12/22	12/11	11/29	12/18	12/15	
01/04			01/01			12/29			

Table 3.6.- **EXAMPLE OF AN ASTRONOMICAL LUNISOLAR CALENDAR.** The columns show the dates of the true astronomical new Moons, on the prime meridian, of the corresponding year (expressed in day/month). The criterion adopted to introduce a thirteenth month is that the start date of the third month (March) is after the spring equinox. If this were not the case, the third month would be embolismic and would not have numbering. The dates in bold correspond to the third month whose beginning is before the spring equinox. These months are the embolismic ones, and therefore, in the years in which they occur, they



Illustracion 1.6.- **AVERAGE DISPERSION OF THE SEASONS IN THE METON CYCLE.** The seasons do not remain fixed on the same dates of the lunisolar cycle. The graph shows the average displacement in days of the seasons according to the year of the Meton cycle, concerning the date the reference season occurred at the beginning of the cycle. Note that at the beginning of the graph, the dispersion is null since we chose, as a reference, the date on which a certain season occurs. In the end, the dispersion is -0.392 days, which is the deviation produced by the inequality between the calendar year and the seasonal year and which accumulates cycle after cycle. The maximum dispersion of a reference season occurs in year 8 of the cycle when the season moves 28.26 days, concerning where it occurred at the beginning of the cycle. The horizontal solid line corresponds to the mean value of the dispersion. The dashed lines are the maximum and minimum averages of the dispersion. Note that the maximum dispersions occur in years 8 and 16 of the cycle, and generally the year before an embolism, and the minimum corresponds to the embolismic years.

And the 6,940-day cycle has the characteristics

$$A = 19; \quad L = 235; \quad d = 6940$$
$$E = 7; \quad m_{30} = 125; \quad m_{29} = 110$$
$$y_m = 365^{\mathrm{d}}.263\,158; \quad l_c = 29^{\mathrm{d}}.531\,914\,89$$
$$M_{30} = 118; \quad M_{29} = 110$$
$$\Delta = 8; \quad \Delta' = 15; \quad S = -4.$$

As regards the rule of intercalation of embolisms and the alternation of months of 29 and 30 days, we apply the rules set out above.

Astronomical lunisolar calendar

In the previous applications of the Meton cycle, we assumed a cycle of 19 years and 235 lunations. However, it is possible to establish an astronomical lunisolar

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are thirteen months long. The table corresponds to 19 years, 235 lunations and 6,940 days. 7 of the years are embolismic. The calendar in the table stabilises the spring equinox date, which always occurs in the third month of the year since we do not number the embolismic months. In the next 19-year cycle, the new Moons will be repeated approximately on the same dates. The embolismic years are in positions 1, 3, 6, 8, 11, 14, and 17, which does not correspond to any of the criteria in Table 2.6, although very close to criterion $\beta = 8$.

calendar and verify that it conforms to the Meton cycle.

Such a calendar has lunar months, for example, beginning on the day of the astronomical new Moon. Moreover, it requires by some method the approximate stabilization of the seasons, for example, by requiring that the date of the astronomical new Moon of the third month of the cycle always be after the astronomical spring equinox. If this were not the case, then that third month would be the embolismic month and would not be counted, the following month being the third, and completing all the years with the 12th month. In this way, the embolismic years would have 13 months, but the third month would be the embolismic month and would not be numbered.

Devising other lunisolar calendars with other characteristics is possible, although we would also recover the Meton cycle. We remember that the new Moons are astronomical and, therefore, have variable durations, whose dates have irregularities. Table 3.6 shows the dates of the new Moons in the 19 years, verifying that there are 7 embolisms and 6,940 days, that is, those that correspond to the Meton cycle.

The techniques discussed in this chapter pertain specifically to computational lunisolar calendars. These calendars operate based on fixed rules and do not rely on observation or astronomical calculations. Consequently, they do not apply to the Chinese calendar, which is an astronomical calendar. However, it is worth noting that over 2,500 years ago, the Chinese calendar used the Meton cycle as a computational method, incorporating seven intercalary months every 19 years.

In Chapter 13, we will examine the alignment of the Chinese calendar with the Meton cycle, which is unsurprising since this cycle is based on the mean astronomical movements of the Moon and Sun.

7.- Solar and lunisolar calendars

Julian and Gregorian calendars

In 46 BCE, Julius Caesar, as Pontifex Maximus of Rome, reformed the previous calendar of the Republican era, thanks to the technical knowledge of the Alexandrian astronomer Sosigenes.

The calendar was named Julian and lasted for sixteen hundred years, proof of how well it was constructed. With minor modifications, it is still in use. The Julian calendar is solar; it completely ignores the Moon and seeks to ensure that the astronomical seasons occur more or less on the same calendar dates. The Julian calendar is computational and prepared without calculation or astronomical observation. Therefore, the Julian calendar is not associated with a specific meridian and can be used interchangeably anywhere on the planet.

The Julian calendar year has an average length of 365.25 days, accomplished through a four-year cycle: three years consist of 365 days each, while one year contains 366 days *, which includes an additional day known as the leap day, added after February 28 **.

The internal structure of the calendar originates in the ancient pre-Julian calendar. It is made up of twelve months of irregular lengths: there are months of 28, 29, 30, and 31 days. The names of the months are those of the Republican calendar, except for the months of July and August, which owe their names to Julius Caesar and Emperor Augustus, respectively. These months were previously called the *quintilis* and *sextilis*. Of the remaining names, some refer to Roman deities, and others refer to a numerical order.

An essential element of the Julian calendar is the seven-day week, although it was not originally part of the Roman calendar. The Western week, which has spread

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* Leap years are those of the Christian era (*Anno Domini*) divisible by 4. This circumstance was not sought but was accidental, given that the Christian era was established in the 6th century and the rule for placing leap years dates back to 46 BCE.

** The placement of leap day is a modern convention. Initially, the leap day was observed after February 24, and that date was repeated. The Christian church continues to follow this custom in its liturgical calendar, where February does not include a 29th day in leap years, meaning February always ends on the 28th. Recently, new legislation was passed in the European Union, establishing February 29 as the standard date for leap day across all member states.

throughout the world, is a mixture of the astrological week, associated with the seven astronomical deities, and the Jewish week, adopted by Christians. The current week is the astrological one that came to Rome from Egypt before Christianity, which is given a special day, Sunday or the Lord's Day, which coincides with the first day of the Jewish week, commemorating the resurrection of Jesus Christ.

The week is a chronological system independent of the calendar. Although once a date in the Julian calendar is known, we can determine its weekly day, the calculations are cumbersome, and in practice, the week runs independently of the calendar; this means that we need a double chronological record to correctly locate a day, needing to know the day of the month and the day of the week.

We want to point out a recent change in the weekly period. Traditionally, Sunday has been considered the first day of the week, a practice followed by Jews and the Christian Church. However, the current civil practice is to take Monday as the first day of the week, as economic logic demands, and the European Union has legislated in this regard.

As we have indicated, computational calendars use astronomical mean values; in particular, the Julian calendar uses an estimate of the tropical year, which causes two problems: the mean length of the astronomical tropical year varies while the calendar year is fixed, and mainly, the mean value adopted in the calendar is a rational number, which cannot be equal to the astronomical value which is an irrational number. Therefore, there is always a difference between the mean value of the calendar and the astronomical value.

For the above reasons, the computational calendars must be corrected to avoid displacements of the seasons on the calendar.

The average length of a Julian calendar year is 365.25 days, while the average astronomical value of the tropical year is 365.2422 days for the year 2000; this means that each year, the Julian calendar year is longer than the astronomical year by just over 11 minutes, accumulating over time. Every 128 years, the seasons will have shifted on average 1 day on the Julian calendar, arriving earlier and earlier.

During the sixteen hundred years that the Julian calendar was in force, there was an accumulated error of approximately 13 days. This shift is slow enough to be noticeable in a person's life. However, Christians celebrate the anniversary of the Resurrection of Jesus Christ on Sunday following the full Moon, which is immediately after or coincides with March 21, the date on which the spring equinox was believed to be fixed. Due to the error pointed out, March 21 was increasingly further away from the spring equinox. So, contrary to tradition, if the Julian calendar had been followed, Easter or the Resurrection of Jesus Christ would have been celebrated not on the first but on the second full Moon of spring.

Due to this circumstance, the Catholic Church was forced to modify the Julian calendar to prevent the spring equinox date from moving too far away from March 21.

There was an excess of leap days in the Julian calendar, which caused its duration to be longer than that of the tropical year. Therefore, it was necessary to reduce the number of leap years. Luis Lilio's proposal was adopted, according to which every 400 years, three leap years are removed, specifically the centennial years (those ending in two zeros) whose first digits are not divisible by four would cease to be leap years. The years 1700, 1800 and 1900 were not leap years despite being

divisible by four, but 2000 was a leap year since 20 is divisible by four.

Since the calendar reform was carried out under the auspices of Pope Gregory XIII, the new rule for intercalating leap years was called the Gregorian rule. This rule made the average length of the calendar year 365.2425 days, a difference of only 26 seconds with the tropical year of 2000, ensuring that the seasons would remain stabilized in the calendar for several thousand years.

The Gregorian calendar was introduced in 1582 in Catholic countries that recognized the authority of the Pope, primarily Spain, France, and Italy. Protestant countries, along with the Orthodox and Anglican churches, initially resisted adopting this papal mandate. However, over time, all Christian nations gradually accepted the Gregorian calendar, along with their overseas territories.

Due to the widespread European influence and the effectiveness of the Gregorian calendar in regulating time, all countries eventually adopted it, leading to its status as the international calendar, even though no formal legislation has been enacted on this issue. Notably, China accepted the Gregorian calendar on two separate occasions, 1912 and 1929.

The Christian era, initially referred to as the era of the Incarnation of Our Lord Jesus Christ, was established in the 6th century by Dionysius Exiguus and adopted by the Christian Church. This chronological system, known by various names, has gained international recognition, although some countries still maintain their traditional systems of chronology *.

Although it is common to refer to the calendar as Gregorian, it is actually still the Julian calendar, with only a slight modification to the rule for intercalating leap years. However, the Christian lunisolar calendar, which we will discuss later, must be called Gregorian because it was the result of a profound change in the calendar that existed before the Gregorian reform.

Another quality of the Julian-Gregorian calendar is that, although leap years were erroneously intercalated when the Julian calendar was implemented, we can trace it backwards with absolute certainty, making it suitable for recording astronomical or historical events.

As we have explained, the Gregorian calendar is excellent from an astronomical point of view. However, it has a cumbersome internal structure of 12 months of unequal lengths, with non-standard names associated with ancient calendar reforms, and where the seasons are not located on special dates. For example, the winter solstice is around December 22, when it would be desirable for it to be January 1. The main defect is the permanent lack of coincidence between the days of the month and the days of the week.

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* The beginning of the Christian era was the moment of the conception of Jesus Christ, which is supposed to have coincided with the spring equinox. However, what was more widespread and what is used today is to count the years from the first day of January, called the Circumcision style. A problem that arises with the computation of the years before the beginning of the era is the use of two systems—the BC, before Christ, and the astronomical system. The difference is that the years before Christ do not have the number 0, which the astronomical method does. Therefore, from the year 1 BC, we move on to the year 1 AD; that is, the year 1 BCE is the astronomical year 0.

Other solar calendars: Indian national and French republican

The successful Gregorian calendar is computational, governed by simple rules rather than by astronomical theory or observation. Numerous proposals have been made to replace the Gregorian calendar, and below, we analyze two of them, which have some elements in common with the Chinese calendar.

In 1793, the calendar established during the French Revolution came into effect; this was a solar calendar consisting of twelve months, each containing 30 days, along with five additional days known as epagomenal or complementary days. What is particularly interesting, especially for the Chinese calendar, is that an astronomical phenomenon determined the structure of the French republican calendar. The year began with the day of the autumn equinox, which was calculated for the meridian of the Paris Observatory, which made the French republican calendar astronomical and, therefore, required the establishment of a reference meridian, which turned out to be the one that passes through Paris.

The attempt to implement the new calendar did not last long; in 1806, it was abolished, and France returned to the Gregorian calendar. The causes of this failure were the excessive ideologisation of the calendar, its local character since it was designed for use in France, and the astronomical criterion for determining the beginning of the year.

By using astronomical theory to determine the beginning of the year, leap years would not occur with the regularity of the Gregorian calendar and, above all, there would be years in which it would not be possible to know with certainty on which day the autumn equinox would occur because it would occur very close to midnight, and it could happen that even astronomical observation would not be decisive, or that due to bad weather it would not be possible to make the observation. The problem we are describing was one of the arguments of those who wanted to return to the Gregorian calendar, which is shared with the Chinese calendar, which uses the winter solstice determined by astronomical theory to regulate the calendar.

In 1957, the Indian government adopted a computational solar calendar that used the Gregorian rule of intercalation. Solar months and their lengths were determined by the average time taken by the Sun to traverse 30 degrees of the ecliptic. The months of the so-called Indian national calendar have 7 months of 30 days and 5 months of 31 days; in leap years, one of the 30-day months is increased to 31 days.

We call the Chinese solar calendar a complementary calendar that serves to prepare the lunisolar calendar. As we will see later (Chapter 11), the Chinese solar months, like the Indian calendar, have months whose duration is the time the true Sun takes to travel 30^{o} of the ecliptic. However, unlike the Indian calendar, the solar calendar does not use average values, and the result is that there are solar months of 29, 30, 31 and 32 days, and not all months have the same duration.

Christian lunisolar calendar

The first Christians wanted to celebrate the death and Resurrection of Christ or Christian Easter annually. They could choose the date in the Julian calendar, but they decided to use a lunisolar calendar similar to the one followed by the Jews at the time of Jesus Christ. According to the Gospels, Jesus Christ died on the 14th or 15th of Nisan, during the full Moon of the first month of spring and was resurrected the following Sunday.

The rule that all Christian churches were to follow in determining the day of commemoration of Jesus' Resurrection was approved at the Council of Nicaea in 325. Easter is the Sunday following the fourteenth day of the Moon that falls on or immediately after March 21. Since the fourteenth day of the Moon (counting the new Moon as day 1) is approximately the full Moon, the rule is that Easter is the Sunday immediately after the first full Moon of spring, assuming, as the early Christians did, that March 21 was the day of the vernal equinox.

The leap day is omitted in Christian *computus* or techniques for finding Easter, because February 24th is repeated in leap years, just like the Romans did. Therefore, February always ends on the 28th, but the corresponding lunar month increases by one day in leap years. So, for the calculation, we assume that the solar year always has 365 days. Therefore, the duration of the Christian Meton cycle is $365 \times 19 = 6,935$ days, and the characteristics of the Christian lunisolar calendar are as follows:

$$A = 19; \quad L = 235; \quad d = 6\,935$$
$$E = 7; \quad m_{30} = 120; \quad m_{29} = 115$$
$$y_m = 365^d; \quad l_c = 29^d.530\,851\,06$$
$$M_{30} = 113; \quad M_{29} = 115$$
$$\Delta = -2; \quad \Delta' = 5; \quad S = 1\ .$$

Noted that in the Christian lunisolar calendar, there is one lunar leap placed in the last year of the cycle. The years that were adopted as embolisms are located in positions 2, 5, 8, 11, 13, 16, and 19, i.e., corresponding to $\beta = 9$ in Table 2.6. The golden number is the order number in the cycle.

The Christian Meton cycle begins in the first year of the Diocletian era, or 285 CE, corresponding to the golden number 1. The calendar attempts to alternate months of 29 and 30 days but only sometimes achieves this due to specific computational rules. The length of lunar years in the Christian Julian lunisolar cycle is 12 of 354 days, 2 of 383, 4 of 384, and 1 of 385, where we assume that the leap day is not counted. If we take the leap day into account, then we would have to increase by one day the length of the year where the leap day falls. Consequently is possible that the Christian calendar also has years of 386 days, and months of 31 days.

The length of the lunar month derived from the Christian lunisolar calendar based on the Meton cycle is somewhat longer than the average astronomical lunation. This error had accumulated since 325, the date of the Council of Nicaea, until 1582, when it was necessary to reform not only the solar calendar but also the lunisolar calendar.

The lunation of the lunisolar calendar exceeds the astronomical lunation by almost 23 seconds, a cumulative amount. The dates of the calendar new Moons at the end of the 16th century came on average four days after the astronomical ones; this means that an error of one day was accumulated approximately every 312 years, in the sense that the astronomical new Moons came on earlier dates than the new Moons marked by the calendar.

The reform of the Christian lunisolar calendar, implemented in 1582, ensured that the calendar months aligned closely with the astronomical months. This required reducing the length of the lunar month. The adopted rule involved removing 8 days from the calendar every 2,500 years, equivalent to eliminating 22 seconds from each

Duration of the lunar year	Frequency	%
353	69 222	10.04
354	167 498	24.29
355	198 736	28.82
383	106 677	15.47
384	36 287	5.26
385	111 052	16.12

Table 1.7 - **FREQUENCY OF THE YEAR LENGTHS OF THE HEBREW CALENDAR**. Frequency of lunar years in the Hebrew calendar according to their length. We calculated for a complete period of 689,472 years and found that the most frequent year is 355 days long, while the least probable is 384 days.

lunation. As a result, the mean lunar month is now only 0.39 seconds longer than the astronomical month, ensuring the stability of the Gregorian lunisolar calendar for several thousand years.

Hebrew calendar

The Hebrew calendar is of the lunisolar type, whose embolismic years are determined by the Meton cycle. The complementary data to construct the cycle is the average duration of the lunation, established at 29.53059375 which differs by just over one second from the average astronomical lunation of the year 2000. Thus, the average duration of the Meton cycle is

$$235 \cdot 29^{d}.530\ 593\ 75 = 6\ 939^{d}.689\ 531,$$

since the cycle must have an integer number of days, the above result means that different Meton cycles must exist depending on the number of days. Embolismic years occur in positions 3, 6, 8, 11, 14, 17 and 19 of the cycle, corresponding to $\beta = 11$ in Table 2.6.

For religious reasons, the Hebrew calendar can delay the beginning of the year by one or two days, resulting in six possible lunar year lengths: 353, 354, 355,

Duration of the lunar year	Frequency	%
6 939	17 099	47.12
6 940	13 648	37.61
6 941	5 246	14.46
6 942	295	0.81

Table 2.7.- **FREQUENCY OF HEBREW METON CYCLE LENGTHS**. The most probable duration of the lunar cycle is 6,939 days; it is unlikely to be 6,942. We have calculated the results of the table for 689,472 years.

Year	Cycle's year	Spring equinox date		Hebrew calendar date
2016	**19**	20	March	10 - Veadar
2017	1	20	March	22 - Adar
2018	2	20	March	4 - Nisan
2019	**3**	21	March	14 - Veadar
2020	4	20	March	24 - Adar
2021	5	20	March	7 - Nisan
2022	**6**	20	March	17 - Veadar
2023	7	21	March	28 - Adar
2024	**8**	20	March	10 - Veadar
2025	9	20	March	20 - Adar
2026	10	20	March	2 - Nisan
2027	**11**	21	March	12 - Veadar
2028	12	21	March	23 - Adar
2029	13	20	March	4 - Nisan
2030	**14**	20	March	15 - Veadar
2031	15	21	March	26 - Adar
2032	16	20	March	8 - Nisan
2033	**17**	20	March	19 - Veadar
2034	18	20	March	29 - Adar
2035	**19**	21	March	10 - Veadar

Table 3.7.- **DISPERSION OF THE SPRING EQUINOX IN THE HEBREW CALENDAR.** We show the dates of the Hebrew calendar in which the spring equinox occurs in the Meton cycle that begins in 2017, taking as the beginning of the day 6:00 p.m. The numbers of the Meton cycle in bold in the second column indicate embolismic years. Veadar or Adar II is the embolismic month and is the month before Nisan. In common years Adar has 29 days and in embolismic years Veadar also has 29 days. Underlined are the dates of the Hebrew calendar, which show that the 15th of Nisan occurs on the second full Moon of spring, assuming the full Moon is on the 14th day of the lunar month. We note that the Nisan full Moon does not occur on any occasion during the winter season.

383, 384, and 385 days, just like the chinese calendar. The intricate Hebrew calendar has a cycle of 689,472 years, after which the calendar repeats itself with the same characteristics. Calculating the frequency of the different lunar years in the previous cycle, we find the results in Table 1.7.

Analyzing the Hebrew calendar in a cycle of 689,472 years, we find four possible lengths of 19-year periods: 6,939, 6,940, 6,941, and 6,942 days, which are the four

Year	Cycle's year	Spring equinox date		Hebrew calendar date
895	**19**	20	March	16 - Veadar
896	1	20	March	28 - Adar
897	2	20	March	9 - Nisan
898	**3**	20	March	18 - Veadar
899	4	20	March	29 - Adar
900	5	21	March	13 - Nisan
901	**6**	21	March	22 - Veadar
902	7	21	March	4 - Nisan
903	**8**	21	March	15 - Veadar
904	9	20	March	25 - Adar
905	10	21	March	8 - Nisan
906	**11**	21	March	17 - Veadar
907	12	21	March	29 - Adar
908	13	20	March	10 - Nisan
909	**14**	21	March	21 - Veadar
910	15	21	March	2 - Nisan
911	16	21	March	14 - Nisan
912	**17**	20	March	24 - Veadar
913	18	21	March	6 - Nisan
914	**19**	21	March	15 - Veadar

Table 4.7.- **SPRING EQUINOX DISPERSION AT THE BEGINNING OF THE 10TH CENTURY**. We find that the 15th of Nisan always occurs in the first month of spring, immediately after the full Moon, which we assume occurs on the 14th of the lunar month.

lengths that the Jewish Meton cycle can have. The frequency of these four lengths is in Table 2.7. Note that each of the four possible lengths of the Meton cycle has different characteristics. The Jewish Meton Cycles of 6,939 and 6,940 days are similar to the two cycles associated with the Julian year but are not identical.

We observe that in the Jewish Meton cycle the duration of the lunation is used as complementary data, therefore the average duration of the solar year is

$$y_m = \frac{Ll_c}{A} = \frac{235 \cdot 29^{\mathrm{d}}.530594}{19} = 365^{\mathrm{d}}.246\ 8205,$$

which is 6 minutes and 13 seconds longer than the Gregorian calendar year which is very close to the tropical year. Here lies the main deficiency of the Jewish Meton cycle, in that although the lunation of the cycle is very close to the astronomical

mean lunation, the length of the solar year deviates significantly from the astronomical value. Every 231 years, the difference with the Gregorian calendar increases by one day; since in this calendar, the seasons remain approximately on the same dates on average, the seasons shift by one day from the dates of the Jewish calendar every 231 years.

On average, the length of a common year corresponds to 12 lunations, and the embolismic year has an average length of 13 lunations

$$\overline{Y}_n = 354^{\mathrm{d}}.367\,125; \quad \overline{Y}_e = 383^{\mathrm{d}}.897\,718,$$

we find the average dispersion of the seasons in the Hebrew calendar with these values. The dispersion increases by $Y_t - \overline{Y}_n$ days in normal years and by $Y_t - \overline{Y}_e$ days in embolismic years. The maximum accumulated dispersion in the Hebrew calendar is 26.35 days, which occurs in year 16 of the cycle. The average dispersion in a cycle is 12.22 days, and the root mean square deviation is 8.69. At the end of the cycle, the deviation is $-0^{\mathrm{d}}.082 = -1^{\mathrm{d}}\,58^{\mathrm{m}}$; this is the secular deviation of the Hebrew calendar

$$\left(365.2425 - 365.246\,8205\right)\cdot 19 = -0^{\mathrm{d}}.082,$$

it is caused by the shorter duration of its solar year in relation to the tropical year. This variation is secular in the sense that it accumulates cycle after cycle and is responsible for the shifting of the seasons by the Hebrew calendar, which reaches one day every 232 years $(\approx 19/0.082)$, in the sense that the Hebrew date of the seasons occurs at earlier dates.

In Tables 3.7 and 4.7, we show the dispersion of the date of the spring equinox in the Hebrew calendar during two Menton cycles beginning in 2017 and 896. At the end of the 9th century, the 15th of Nisan, the Passover holiday, always occurs immediately after the equinox, that is, during the first full Moon of spring. However, as we see in Table 3.7, at the beginning of the 21st century, there are years in which the 15th of Nisan occurs on the second full Moon of spring. In both tables, we see that the Passover is not celebrated at the end of winter.

By comparing Tables 3.7 and 4.7, we see that over time, the spring equinox occurs at earlier dates in the Hebrew calendar as a result of the different lengths of the calendar-derived year and the tropical year.

Computational calendars, unlike astronomical or observational calendars, require periodic corrections. The Christian lunisolar calendar has a self-regulating mechanism that ensures its accurate functioning for several thousand years. However, the Hebrew calendar lacks this mechanism. Therefore, some form of correction will be necessary to maintain its proper functioning in the future.

8.- Rules of the Chinese calendar

The rules of the Chinese calendar have changed many times. The system we have today dates back to 1645, when Jesuit missionaries proposed that, like the lunar coordinates, the solar coordinates used to compile the calendar should be true and not mean, as had been the case until then.

In 2017, the Chinese government released new regulations regarding the calendar. Below, we outline the fundamental rules in greater detail than what is provided in the official guidelines The concepts that appear in the following text have been or will be developed in other chapters.

Introduction

1.- In China, the promulgation of the calendar is an important governmental matter.
2.- The Purple Mountain Observatory of the Chinese Academy of Sciences is responsible for compiling the calendar.
3.- The rules for preparing the calendar are established in document GB/T 33661-2017, jointly published by the General Administration of Quality Supervision, Inspection and Quarantine of the People's Republic of China and the Standardization Administration of the People's Republic of China, drafted by the Purple Mountain Observatory.
4.- The astronomical theory needed to create the calendar is separate from the calendar's regulations. While the rules do not dictate which astronomical theory to use, they impose certain limitations. They require that the accuracy of astronomical calculations be within 1 second when determining the winter solstices and newMoons.

Primary rules

5.- Similar to civil time, days start and end at midnight.
6.- Astronomical calculations reference the meridian at 120º E, corresponding to UTC+8 hours.

> *Note 1.*- UTC is the coordinated universal time and is the civil time scale referred to the prime meridian.

7.- Lunar months start at 0 hours on the day of the geocentric new Moon, regardless of when this astronomical event occurs.

> *Definition 1.*- The ecliptic longitude of the Sun or the Moon is the angle measured along the ecliptic in the direction of the Sun's or Moon's proper motion. This

angle is measured from the true equinox of the date (which considers factors such as precession and nutation) to the Sun's or Moon's centre. It is calculated from the Earth's centre (geocentric longitude) and corrected for aberration and light-time. Therefore, it represents an apparent longitude rather than a geometric one.

Definition 2.- We define the geocentric new Moon or conjunction of the Sun and the Moon as the moment when the apparent geocentric ecliptic longitudes of the centers of both celestial bodies are equal.

Note 2.- Note that the conjunction is a geocentric and not a topocentric phenomenon; it occurs simultaneously in all places on Earth, although they are expressed at different times if we use local times.

8.- The lunar months are named in numerical order, from 1 to 12.

9.- The embolismic, abundant, thirteenth or intercalary lunar months of the lunar year take the number of the preceding month by placing the word intercalary in front.

Note 3.- Intercalar months can take all possible names from intercalary-1 to intercalary-12.

10.- Lunar month 11 is the month in which the winter solstice occurs, calculated according to the apparent geocentric position of the Sun's centre.

Definition 3.- The winter solstice occurs when the Sun's apparent geocentric ecliptic longitude is 270º.

Note 4.- The winter solstice is when the Sun reaches the lowest possible declination; therefore, when it passes through the meridian, it projects the largest shadow of the gnomon.

Note 5.- The winter solstice is a geocentric phenomenon, meaning it occurs simultaneously at all locations on Earth. However, when considering local time, the solstice will be observed at different times in different places.

11.- If the *sui* has 13 lunar months, one of them is the thirteenth month or intercalary month.

Definition 4.-The *sui* is the period formed by the lunar months that go from the end of the 11th month of the previous year to the end of the 11th month of the current year.

Definition 5.- The true solar year in the Chinese calendar is defined as the period between two consecutive winter solstices. The mean solar year refers to the average length of these true solar years.

Note 6.- The *sui* is one of two methods for defining the lunar year, which is distinct from the solar year.

Note 7.- A solar year consists of 12 lunar months and parts of one or two additional lunar months. A solar year can never have thirteen complete lunar months.

12.- The intercalary lunar month is the first month of the *sui* period which does not contain a major solar term.

Definition 6.- The Sun's ecliptic orbit is divided into 24 sections, each covering 15° of longitude. The moments when the apparent geocentric ecliptic longitude of the Sun's center is a multiple of 15° are known as solar terms. For calendar purposes, solar terms begin at 0 hours, regardless of the specific time of day

when they occur.

Note 8.- The solar terms generalise the equinoxes and solstices, which divide the solar ecliptic orbit into sections of 90º each.

Note 9.- Each solar term is given a name related to the climatology of central China (13 of them), with the seasons (8 of them), and two are agricultural names (Table 1.8).

Definition 7.- Major solar terms or *zhongqi* are the positions of the Sun when its apparent geocentric ecliptic longitude is $300 + 30 \times n$, where n is an whole from 1 to 12. We identify the major solar terms with the letter Z followed by a number from 1 to 12; that is, Z1, Z2,... and Z12. The major solar term Z1 has a solar longitude of 330º corresponding to $n = 1$.

Definition 8.- Minor solar terms or *jieqi* are the positions of the Sun when its apparent geocentric ecliptic longitude is $315 + 30 \times (m - 1)$, where m is an whole from 1 to 12. We identify the minor solar terms by the letter J followed by a number from 1 to 12; that is, J1, J2,..., and J12. The minor solar term J1 is that of solar longitude 315º, corresponding to $m = 1$.

Definition 9.- The period between two consecutive New Year's days is called *nian*.

Note 10.- *Nian* and *sui* do not coincide, although the two periods overlap by 11 or 12 lunar months. *Nian* and *sui* are two ways of defining the lunar year, and each differs in the time each type of year starts.

Definition 10.- New Year's day (or Spring Festival) is on the 1st day of the 1st month.

Definiton 11.- We assume that the *nian* lunar year corresponds to the AD year (*Anno Domini,* or year of the Christian era or Gregorian year), whose New Year's day, or the first day of the year, is closest to January 1. The *sui* lunar year corresponds to the AD year, whose first day (or day 1 of the month following the 11th month) is closest to January 1.

Note 11.- The *nian* and *sui* always occur between two Gregorian years (at least in the present day).

13.- The second month after the eleventh lunar month (excluding intercalary months) is the start of the *nian* year, or month 1, whose first day is the New Year.

Note 12.- Although unlikely, it is possible that intercalary month-11 and intercalary month-12 exist, in which case month 1 or the start of the *nian* lunar year is the third month after month 11.

Note 13.- The fact that there are two months between month 11 of the previous year and month 1 of the current year does not mean there will be a one-month delay at the start of the new year.

Secondary and derived rules

14.- Solar terms start with J1 and end with Z12.

15.- Solar term J1 (corresponding to solar longitude 315º) is the midpoint between the winter solstice (Sun longitude 270º) and the spring equinox (Sun longitude 0º). Solar term J1 is the start of the Chinese spring.

16.- Solstices occur when the Sun is at Z5 (summer solstice) and Z11 (winter solstice). Equinoxes occur when the Sun is at Z2 (spring equinox) and Z8 (autumn equinox).

17.- Each of the 24 solar sections lies between a minor solar term and a major solar

Solar term	Name		Sun's longitude	Gregorian date
J1	*Lichun*	Beginning of Spring	315	February 3 or 4 or 5
Z1	*Yushui*	Rain Water	330	February 18 or 19 or 20
J2	*Jingzhe*	Waking of insects	345	March 5 or 6 or 7
Z2	*Chunfen*	Spring Equinox	0	March 20 or 21 or 22
J3	*Qingming*	Pure Brightness	15	April 4 or 5 or 6
Z3	*Guyu*	Grain Rain	30	April 19 or 20 or 21
J4	*Lixia*	Beginning of summer	45	May 5 or 6 or 7
Z4	*Xiaoman*	Grain full	60	May 20 or 21 or 22
J5	*Mangzhong*	Grain in ear	75	June 4 or 5 or 6
Z5	*Xiazhi*	Summer solstice	90	June 20 or 21 or 22
J6	*Xiaoshu*	Sligth heat	105	July 6 or 7 or 8
Z6	*Dashu*	Great heat	120	July 22 or 23
J7	*Lliqiu*	Beginning of autum	135	August, 6 or 7 or 8
Z7	*Chushu*	Limit of heat	150	August 22 or 23 or 24
J8	*Bailu*	White dew	165	September 7 or 8
Z8	*Qiufen*	Autumnal equinox	180	September 22 or 23 or 24
J9	*Hanlu*	Cold dew	195	October 7 or 8 or 9
Z9	*Shuangjiang*	Descent of frost	210	October 22 or 23 or 24
J10	*Lidong*	Beginning of winter	225	November 6 or 7 or 8
Z10	*Xiaoxue*	Slight snow	240	November 21 or 22 or 23
J11	*Daxue*	Great snow	255	December 6 or 7 or 8
Z11	*Dongzhi*	Winter solstice	270	December 21 or 22 or 23
J12	*Xiaohan*	Slight cold	285	January 4 or 5 or 6
Z12	*Dahan*	Great cold	300	January 19 or 20 or 21

Table 1.8.- **SOLAR TERMS**. The table shows the twenty-four solar terms into which the Sun's ecliptic path is divided. J corresponds to the minor terms and Z to the major terms. In the fifth column are the Gregorian dates of the solar terms from the 17th to the 23rd century according to the Beijing time zone and expressed in Coordinated Universal Time (UTC+8). Because of the Gregorian rule of intercalation of leap years, the years in which solar terms arrive earliest in the Gregorian calendar are the years 1696, 1796, 1896, 2096, ..., i.e. the beginning of the four years before a centennial year where the leap day is omitted (1700, 1800, 1900, 2100, etc.) and the years in which solar terms arrive latest in the Gregorian calendar are 1703, 1803, 1903, 2103, ..., i.e. the years preceding leap years that immediately follow non-leap centennial years. Some of the earlier dates will not occur again in the future, while some of the later dates have not yet occurred.

term (J1-Z1, J2-Z2, ...).
18.- Lunar months can only be either 29 or 30 days long. The months that are 30 days long are referred to as long months, or *dayue*, while those that are 29 days long are called short months, or *xiaoyue*. Long months occur slightly more frequently than short months.
19.- Month 11 cannot be intercalary because it always contains the major solar term of the winter solstice or Z11. However, an intercalary month-11 can exist when the month following the 11th is an intercalary month. Month 11 can have two major solar terms.
20.- The *sui* year may be intercalary (i.e. it has 13 lunar months), but the *nian* year is common (12 lunar months) and vice versa. However, it is most likely that when there is a common *sui*, the corresponding *nian* will also be common, and the same for embolismic years.
21.- When there is an intercalary *sui* (i.e. 13 lunar months), the intercalary month may belong to the current *nian* or very unlikely to the previous *nian*.

Astronomical calculations

22.- The astronomical theory used to calculate the Chinese calendar is not part of the calendar rules; that is, it does not require using a specific astronomical theory but must meet the following conditions.
23.- Astronomical concepts are defined using terminology defined by the International Earth Rotation and Reference Systems Service (IERS) conventions.
24.- The times of the new Moons and winter solstices correspond to true astronomical values, not mean values. Therefore, the calendar uses the synodic (true, not mean) month or time between two consecutive new Moons and the winter seasonal (true, not mean) year or time between two consecutive winter solstices.
25.-The astronomical calculation for determining the new Moons and winter solstices must be accurate to within 1 second, which means 0.04 arc seconds for the longitude of the Sun and 1.25 arc seconds for the longitude of the Moon.
26.- The accuracy of the astronomical calculation does not account for intercalary seconds that have not been officially published by the time the calendar is prepared.

Miscellany

27.- Usually, lunar months have two solar terms, one major and one minor. However, after the last calendar reform, winter months can have up to three solar terms (two of one type and one of the other type); this can only occur in months 10, 11, 12 and 1.
28.- A lunar month usually has a major and a minor solar term, but the order in which they occur within the month can vary.
29.- Usually, a lunar month without a major principal term is an intercalary month.
30.- If in a non-intercalary *sui* year, a lunar month has two major terms, there will be a lunar month without a major term, which does not always belong to the same *sui* year; this will occur in the solar year corresponding to the Gregorian year 2033 when month 11 will have two major terms and month 8 had no major term. This strange circumstance also occurred in 1985 when the first lunar month had no major term; none of the lunar months of that year had two major terms. However, month 12 of the previous year did.
31.- The Gregorian calendar new year almost always falls in month 11 or 12 (2033

was an exception because it will fall on intercalary-11).
32.- The Chinese seasons correspond to the minor solar terms J1, J4, J7, and J10, while the Western seasons correspond to the major solar terms Z2, Z5, Z8, and Z11.
33.- Except in exceptional cases, the solar term Zn is in the lunar month n. In the calendar prior to the reform in the mid-17th century promoted by the Jesuits, the solar term Zn always coincided with the lunar month n; that is, there could not be two major solar terms in a lunar month, and therefore, a non-intercalary lunar month, could not exist without a solar term *.
34.- Non-intercalary *nian* (or *sui*) years can be 353, 354, and 355 days; embolismic years, whether *nian* or *sui*, can have lengths of 383, 384, and 385 days.
35.- Chinese New Year's day can occur between January 21 and February 21 of the Gregorian calendar, both inclusive.
36.- Since the intercalary month has no major solar term, the number of major terms usually corresponds to the number of the lunar month in which they are found. This rule is violated in lunar months with two major solar terms.
37.- The lunar month containing Z1 typically is the first month of the lunar year. Only in sporadic cases is Z1 the solar term of the 12th month, which occurs when the 11th or 12th month has two major solar terms; this is what happened in 1985 when Z1 or *Yushui* was the 30th of the 12th month of the previous lunar year, as a result of the 11th month of that year having two major terms.
38.- The rule for knowing when a month is intercalary is called the non-*zhongqi* rule.

Sui and *nian* lunisolar years

The Chinese calendar is lunisolar, meaning that its months are lunar. The first day coincides with the astronomical new Moon, and the year can have 12 or 13 months. The date of the winter solstice is used to determine whether the year is normal or abundant.

There is one more thing: the rules of the Chinese calendar allow us to determine the position of the intercalary or embolismic month in the year, which is not the case in other lunisolar calendars, whose intercalary months are predefined and their positions in the year are invariable. Therefore, in a certain sense, the Chinese calendar is a combination of a solar calendar and a lunar calendar; while the former determines the position of the Sun in its path, the latter determines the lunar months.

The lunisolar year, or *nian*, starts with the first day of the first month, with the day of the Chinese New Year. If the *nian* year has 13 months, it is an intercalary year; if it has 12 months, it is a common year. However, we can only determine the date of the New Year if we know the position of the intercalary month in the previous year (if there was one). If we do not know when the *nian* year starts, we cannot know if it will have 12 or 13 months. Therefore, we cannot use the *nian* year to determine if the

* In the calendar before 1645, the mean motion of the Sun was used, which describes the orbit at a constant velocity. The time the mean Sun takes to pass through two consecutive major solar terms is approximately 30.44 days, a period that does not fit into a lunar month with a maximum duration of 30 days. For a non-intercalary lunar month without a solar term to exist in the old calendar, it would be necessary to have, to compensate, a solar month with two solar terms, which we have seen was not possible.

year is embolismic **.

To determine if the year is intercalary, we use the property that the 11th month is where the winter solstice occurs. The first operation to prepare the Chinese calendar is to find the lunar months in which the winter solstices of the current year, the previous year and the following year occur. Then, we check the months between the consecutive 11th months, and we can find out if they are 12 or 13 lunar months. As we have said, the period between two 11th months, specifically from the end of one 11th month to the end of the following 11th month, is called a *sui* lunisolar year.

The term *sui* is usually associated with the astronomical year, which we define in two possible ways: as the time between two consecutive winter solstices or as the interval between the days in which two consecutive winter solstices occur. By the first definition, we find a decimal number; by the second definition, we find an integer.

In this book, we have a different definition of a *sui* lunisolar year than usual, and we understand it as the months that elapse between two consecutive months after the 11th. Note that the month after the 11th is not always the 12th because, in some exceptional circumstances, it can be the intercalary-11 month. While the astronomical *sui* year (that is, the time between two winter solstices) has a duration close to the tropical year, the *sui* lunisolar year (as we have defined it before) has a variable duration, as occurs in lunisolar years, with a minimum duration of 353 days and a maximum of 385 days.

Nian and *sui* years are lunisolar, meaning they sometimes have 12 and sometimes 13 months. They differ from each other in the time at which they begin. While the *nian* year begins on New Year's day, the *sui* year begins one or two months earlier, coinciding with the first day of the month following the 11th month. Although it is very rare, the *sui* year may have 13 months and the *nian* year 12 months and vice versa. It is usual for both years to have the same number of months.

From the above we conclude that to know if a *nian* year is embolismic, we need to know the associated *sui* year and the *sui* year after the *nian* year we are considering. The above considerations will be treated descriptively in Chapter 10.

Calendar meridian

The Chinese calendar is astronomical because we need to know two astronomical phenomena to prepare it: the new Moons and the winter solstices. Both phenomena are geocentric; they occur simultaneously in all places, unlike topocentric phenomena that depend on the observer's location on the Earth's surface.

When calculating a geocentric astronomical phenomenon, we need to indicate which meridian, or rather the time zone, must be used to express the time of that

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** If there were no intercalary months between the start of the *sui* year and the start of the *nian* year, then the *nian* year would always start in the second lunar month after the 11th month, which is defined because the winter solstice occurs in that month. If this were so, the New Year's date would be directly determined by the 11th month. However, although it is improbable, it is possible that there are intercalary months 11 and 12, so the first month of the *nian* calendar would be the third month after the 11th month. Since we do not know in advance whether there is an intercalary month 11 or 12, we cannot define the New Year exclusively by the 11th month.

phenomenon; this means that astronomical calendars need a reference meridian; this is not the case with computational calendars, which are governed by fixed rules not linked to an astronomical phenomenon, as is the case with the Gregorian calendar, which has no associated meridian. However, for all calendars, the day is governed by local civil time; therefore, they depend on the time zone.

Since the calendar has a meridian to express time, it is no longer universal, as are computational calendars, and it is not easy to accept it in places far from the reference meridian. This problem was noted with the calendar used during the French Revolution at the end of the 18th century. That calendar began the new year with the autumn equinox according to the time of the Paris meridian, which, together with other local calendar characteristics, would have made its application to other territories difficult.

During the Qing Dynasty (1644-1911), astronomical calculations for preparing calendars were based on local solar time in Beijing. From 1924 to 1928, the mean time of the same meridian was utilized. Beginning in 1929, the time corresponding to the 120^{o} E meridian was adopted for calendar purposes, which is the method still in use today. As a result, the Chinese calendar operates on a scale of UTC+8 hours. It is important to note that if we were to select the time of a different meridian, we would produce a different calendar.

Astronomical new Moons and winter solstices can occur at any time of day. However, in the calendar, we use the day these astronomical phenomena occur, which starts at 0 hours according to the Beijing time zone.

Solar terms

As we have explained in the primary rules of the calendar, the Sun's movement across the ecliptic is divided into 24 sections of 15^{o} each. The Sun can reach these solar terms at any time of the day. In Table 2.8, we give, as an example, the moments when the solar terms occur in the year 2025. Due to the intercalation of leap years in the Gregorian calendar and the irregularities of the Sun's movement, there is a deviation from year to year in the dates of the solar terms. The major solar terms are of interest in the calendar, which are 30^{o} apart.

The time the Sun takes to travel through each solar term varies since the velocity of the Sun in its orbit depends on its distance from the Earth, being maximum at perihelion and minimum at aphelion. In Table 3.8, we have calculated the times taken by the Sun to travel through the intervals between solar terms in the year 2025. We see that the maximum duration is 31.449 days, which occurs in the solar term that begins with the summer solstice, and the shortest duration is 29.444 days, which appears in the most extended solar term of the winter solstice.

As we have said, the solar terms of the calendar start at 0 hours of the day in which they occur, so we identify the solar term by a day of the lunar month and not by an instant. Table 4.8 shows the durations between the days of two consecutive major solar terms, which we will later call solar months (Chapter 11).

Critical dates

The official rules of the Chinese calendar require that calculations for new Moons and winter solstices be accurate within 1 second, excluding any intercalary seconds that may not have been announced. However, it seems illogical to demand

Solar terms	Gregorian date	Time UTC+8
J1	2025/02/03	22:10:12
Z1	2025/02/18	18:06:36
J2	2025/03/05	16:07:16
Z2	2025/03/20	17:01:27
J3	2025/04/04	20:48:40
Z3	2025/04/20	03:56:05
J4	2025/05/05	13:57:18
Z4	2025/05/21	02:54:39
J5	2025/06/05	17:56:34
Z5	2025/06/21	10:42:17
J6	2025/07/07	04:04:59
Z6	2025/07/22	21:29:22
J7	2025/08/07	13:51:25
Z7	2025/08/23	04:33:49
J8	2025/09/07	16:51:55
Z8	2025/09/23	02:19:15
J9	2025/10/08	08:41:01
Z9	2025/10/23	11:50:46
J10	2025/11/07	12:03:57
Z10	2025/11/22	09:35:28
J11	2025/12/07	05:04:28
Z11	2025/12/21	23:02:51
J12	2026/01/05	16:23:02
Z12	2026/01/20	09:44:51
J1	2026/02/04	04:02:07
Z1	2026/02/28	23:51:50
J2	2026/03/05	21:58:52
Z2	2026/03/20	22:45:52
J3	2026/04/05	02:40:01

Table 2.8.- **DATES AND TIMES OF SOLAR TERMS**. Gregorian dates of solar terms for the Gregorian year 2025/2026. The time scale is civil time corrected for Beijing time zone, i.e. UTC+8.

Solar terms	Days between solar terms
Z11 / 2024	29.444
Z12 / 2025	29.588
Z1 / 2025	29.955
Z2/ 2025	30.455
Z3 / 2025	30.957
Z4 / 2025	31.325
Z5 / 2025	31.449
Z6 / 2025	31.295
Z7 / 2025	30.907
Z8 / 2025	30.397
Z9 /2025	29.906
Z10 / 2025	29.561
Z11 / 2025	29.446
Z12 / 2025	29.588
Z1 / 2026	29.954

Table 3.8.- **DURATION OF MAJOR SOLAR TERMS**. The table illustrates the time it takes for the Sun to travel through the major solar terms in 2025. Precisely, it measures the duration required for the Sun to cover the 30 degrees of ecliptic longitude that separate these solar terms. During this period, the longest duration is 31.449 days, which corresponds to solar term Z5, occurring on June 21 (the summer solstice). Conversely, the shortest duration is 29.444 days, in solar term Z11, which began on December 21, 2024 (winter solstice).

such precise accuracy in astronomical calculations when the unpredictability of intercalary seconds can introduce greater errors.

We want to check whether the requirement for a 1-second accuracy for the timing of new Moons and winter solstices is justified. In Table 5.8, we have placed the new Moons that we call critical, those that are less than 5 minutes from 0 hours, either 5 minutes before or 5 minutes after. We found only 27 critical lunations from 2025 to the beginning of the 25th century. We have performed the calculations with the simplified VSOP87 theory, expressing the result in the local time of the Beijing time zone and with UTC(2017), counting only the 27 seconds that have been intercalated until 2017.

Suppose a lunation is very close to midnight. In that case, due to inaccuracies in astronomical theory, the placement of intercalary seconds, or because the civil time scale has been changed, there is doubt whether the new Moon falls on one day or another; this will mean that there is no certainty as to when the lunar month begins.

From the data in Table 5.8, we see that the most critical date for new Moons is

Major solar term dates		Days between major solar terms	Major solar terms
First day	Last day		
2023/12/22	2024/01/19	29	Z11 / 2023
2024/01/20	2024/02/18	30	Z12 / 2024
2024/02/19	2024/03/19	30	Z1 / 2024
2024/03/20	2024/04/18	30	Z2 / 2024
2024/04/19	2024/05/19	31	Z3 / 2024
2024/05/20	2024/06/20	32	Z4 / 2024
2024/06/21	2024/07/21	31	Z5 / 2024
2024/07/22	2024/08/21	31	Z6 / 2024
2024/08/22	2024/09/21	31	Z7 / 2024
2024/09/22	2024/10/22	31	Z8 / 2024
2024/10/23	2024/11/21	30	Z9 / 2024
2024/11/22	2024/12/20	29	Z10 / 2024
2024/12/21	2025/01/19	30	Z11 / 2024
2025/01/20	2025/02/17	29	Z12 / 2025
2025/02/18	2025/03/19	30	Z1 / 2025
2025/03/20	2025/04/19	31	Z2 / 2025
2025/04/20	2025/05/20	31	Z3 / 2025
2025/05/21	2025/06/20	31	Z4 / 2025
2025/06/21	2025/07/21	31	Z5 / 2025
2025/07/22	2025/08/22	32	Z6 / 2025
2025/08/23	2025/09/22	31	Z7 / 2025
2025/09/23	2025/10/22	30	Z8 / 2025
2025/10/23	2025/11/21	30	Z9 / 2025
2025/11/22	2025/12/20	29	Z10 /2025
2025/12/21	2026/01/19	30	Z11 / 2026

Table 4.8.- **DAYS BETWEEN TWO CONSECUTIVE MAJOR SOLAR TERMS**. The table shows the major solar terms for the years 2024-2025. The first column shows the day the Sun passes through the major solar term, and the second column shows the day before the Sun passes through the next major solar term. The table shows the time it takes the Sun to go from one solar term to the next, which we will later call the solar month of the calendar (Chapter 11). We see that the durations can be 29, 30, 31 and 32 days, with the longest periods occurring in the summer months, when the Sun is close to the aphelion where it has a lower velocity and therefore takes longer to travel 30^{o} of the ecliptic.

Critical new Moons		Major solar terms		
2027/02/06	23:56:04	2027/02/19	05:33:31	Z1
2057/09/29	00:00:39	2057/10/23	06:09:17	Z9
2082/07/25	23:55:37	2082/08/22	23:14:41	Z7
2089/09/04	23:59:15	2089/09/22	14:08:21	Z8
2097/08/08	00:01:43	2097/08/22	14:23:51	Z7
2115/02/24	23:59:25	2115/03/21	12:24:33	Z2
2116/05/12	23:59:30	2116/05/21	02:49:02	Z4
2133/09/29	00:02:16	2133/10/23	16:03:27	Z9
2165/12/04	00:01:20	2165/12/21	22:44:07	Z11
2170/10/18	00:01:53	2170/10/23	15:11:33	Z9
2155/06/18	23:55:16	2155/06/21	20:32:32	Z5
2209/10/29	23:58:47	2209/11/23	00:57:05	Z10
2221/05/23	00:03:46	2221/06/21	19:18:43	Z5
2228/07/02	23:58:45	2228/07/22	22:40:39	Z6
2230/07/02	23:56:35	2230/05/21	16:20:26	Z4
2248/10/17	23:58:35	2248/10/23	12:38:29	Z9
2254/09/13	00:01:10	2254/09/23	12:19:40	Z8
2259/11/16	00:01:35	2259/11/23	03:55:49	Z10
2298/06/09	23:55:37	2298/06/21	09:52:11	Z5
2299/02/01	23:57:39	2299/02/19	05:39:15	Z1
2300/02/20	23:57:18	2300/03/21	08:50:18	Z2
2312/05/07	23:57:58	2312/05/21	12:26:07	Z4
2323/04/06	23:55:14	2323/04/21	07:12:35	Z3
2333/02/15	23:59:57	2333/02/19	11:37:29	Z1
2334/10/28	23:57:26	2334/11/23	08:34:14	Z10
2344/01/16	23:56:18	2344/02/20	03:30:11	Z1
2419/08/22	00:01:52	2419/08/23	10:16:58	Z7

Table 5.8.- **CRITICAL NEW MOONS**. The table shows the new Moons from 2024 to the beginning of the 25th century that occur five minutes before or after midnight, which we call critical new Moons. The dates of the major solar terms corresponding to critical lunations are also shown. We can see that an error of a few minutes in the new Moon will not affect the solar terms corresponding to that lunation, which will still belong to the same lunar month. The times in the table correspond to UTC(2017) + 8. UTC(2017) is the Coordinated Universal Time scale for the year 2017, which included 27 intercalary seconds. This calculation accounts for the intercalary seconds added to the end of 2024, resulting in 27 intercalary seconds. Note that we do not anticipate any additional intercalary seconds being added in the future.

September 29, 2057. Since it will be only 39 seconds from midnight, it could belong to the previous day and, therefore, alter the lunar month's start with that lunation.

Table 6.8 shows the solar terms that occur 5 minutes before or after midnight from 2025 until the beginning of the 25th century. There is no particularly critical solar term. Although we cannot determine the exact timing of solar terms, they do not likely influence the calendar, even in the worst-case scenario.

From the above, we conclude that there is some uncertainty in some new Moons and solar terms due to the seconds that will be intercalated in the future or due to alteration of the civil time scale, but the uncertainty is such that it is excessive to demand an astronomical calculation with an accuracy of less than 1 second.

Gregorian calendar and Chinese calendar

To prepare the Chinese calendar, it is necessary to calculate the moments of the new Moons and the winter solstices using astronomical theory. The temporal argument of the astronomical ephemerides is Julian centuries of terrestrial time, from which we deduce the Julian day of the astronomical phenomenon, with which we find the Gregorian date (Appendix III).

We could prepare the Chinese calendar by knowing the Julian days of the new Moons and the winter solstices, calculated from astronomical theory. However, this calendar thus prepared could not be placed in time because by using only the Chinese calendar, we cannot determine the year of the calendar we have prepared. Now, if we associate even one date of the Chinese calendar with the Gregorian calendar, we could already know the year of the Chinese calendar we have prepared. We see then that it is necessary to have the Gregorian calendar, or another computational calendar, to prepare and place the Chinese calendar in time.

Using astronomical theory, we find the Julian days of the new Moons and winter solstices, but we cannot find the Chinese calendar dates by knowing the Julian days. We find the corresponding Gregorian dates and associate them with the Chinese calendar dates.

For this reason, the Gregorian calendar is part of the Chinese calendar since the practice we use is to associate Chinese dates with the Gregorian calendar.

Defects of the Chinese calendar

The Chinese calendar suffers from several defects, which, in our opinion, make it inferior to computational calendars, particularly the Gregorian calendar. Most of the problems of the Chinese calendar come from its astronomical nature; that is, it depends on the calculation of astronomical phenomena, which are also intended to

Critical Major Solar Term			New Moon	
2051/03/20	Z2	23:59:18	2051/03/13	00:53:46
2084/03/20	Z2	00:00:41	2084/03/07	01:05:15
2114/11/22	Z10	23:58:04	2114/10/28	22:03:23
2133/07/22	Z6	23:55:43	2133/07/03	01:14:15
2147/11/22	Z10	23:56:44	2147/11/23	15:46:34
2150/03/21	Z2	00:04:58	2150/02/27	18:51:23
2155/10/23	Z9	23:58:42	2155/09/27	01:24:10
2157/12/21	Z11	23:59:38	2157/12/02	08:16:29
2183/03/21	Z2	00:00:52	2183/02/22	20:12:51
2213/11/22	Z10	23:56:15	2213/11/13	19:01:19
2249/03/20	Z2	23:59:43	2249/03/15	04:10:06
2256/09/23	Z8	00:01:51	2256/09/19	23:20:37
2260/08/23	Z7	00:02:55	2260/08/08	08:01:17
2282/03/20	Z2	23:58:00	2282/03/11	03:37:47
2288/01/21	Z12	00:04:26	2288/01/04	20:33:42
2289/08/23	Z7	00:02:26	2289/08/17	05:37:46
2314/05/21	Z4	23:56:02	2314/05/15	17:24:52
2317/06/22	Z5	00:00:11	2317/06/10	20:24:45
2340/07/22	Z6	23:58:17	2340/06/26	10:03:42
2346/06/22	Z5	00:00:51	2346/06/20	21:01:34
2352/08/23	Z7	23:55:20	2351/08/23	04:24:45
2358/01/20	Z12	23:57:31	2358/01/11	03:36:48
2369/07/23	Z6	00:01:03	2369/07/05	08:01:35
2380/08/22	Z7	23:59:07	2380/08/02	02:12:16
2380/09/22	Z8	23:55:17	2380/08/31	10:56:50
2381/03/20	Z2	23:55:59	2381/02/25	12:23:18
2442/09/22	Z8	23:59:57	2442/09/05	09:23:20

Table 6.8.- **CRITICAL MAJOR SOLAR TERMS**. In the table, we show the critical major solar terms, i.e. those less than 5 minutes from midnight, so there is a possibility that the solar term belongs to one day or another. We also show the times of the new Moons in the months in which we found the previous solar terms. We check that a one-day shift of the solar terms does not affect the calendar, except for the solar term Z10 of the year 2147, which occurs on November 22, 2147, which could belong to one lunar month or the next.
We have not anticipated any new intercalary seconds in our calculations, so we have only considered those implemented until the end of 2017.

be as exact as possible. Below, we detail the defects that we see in the Chinese calendar:

* The Chinese calendar requires astronomical knowledge that the average person does not have; this means that control of the calendar is in the hands of an «authority» outside the population. We are dealing with a calendar that is not «democratic» or popular, in the sense that the population cannot control it.
* Since the Chinese calendar is complex to calculate, errors may occur in its preparation or the software.
* The Chinese calendar cannot be traced backwards with any certainty because of its numerous alterations, either by changes in its rules or by the use of other astronomical techniques.
* We cannot be certain when preparing the calendar for the medium-term future, mainly because intercalary seconds are unpredictable.
* The Chinese calendar would be affected by any change in the civil time scale because it would alter the relationship between the uniform time of Astronomy and the civil time scale.
* The Chinese calendar has no era, meaning that to fix an event in the past, we have to refer to the Western era, which is foreign to the Chinese calendar's tradition.
* With current astronomical techniques, we need a computational calendar to calculate the calendar. Currently, the Chinese calendar uses the Gregorian calendar as an aid.
* There is a reference meridian where Beijing is located; this makes the calendar local and linked to a particular place, which does not occur in computational calendars.
* The words related to the calendar come from Chinese, which makes the calendar even more local.
* The names of solar terms (Table 1.8) refer to the climatology of a certain part of China and are, therefore, not related to other places on the planet.
* There is no uniformity in the internal structure of the calendar. The same month may have some years 29 days and others 30 with an unpredictable rhythm. Any lunar month may be the embolism month, although not all with the same probability. Although it approximately follows the Meton cycle, the sequence of the embolism years cannot be found with a simple mnemonic rule, and we must resort to astronomical calculation to be sure.

Compared to computational calendars, the primary advantage of the Chinese

calendar is that it consistently aligns with astronomical events, eliminating the need for corrections. Additionally, the Chinese calendar plays a significant role in the traditions of several Asian countries. It is a remarkably clever system that tracks the movements of both the Sun and the Moon because its intercalary month is not fixed.

Some changes that we believe would make the Chinese calendar more democratic or accessible to the general public would be to return to the old system of using average values of the lunation and the winter seasonal year or convert it into a computational calendar subject to the Meton cycle with some system of self-regulation.

9.- Sexagenary cycle

Sexagenary cycle

The *nian* lunar years are named according to the sexagesiomal cycle, or *ganzhi*, traditionally composed of 10 heavenly stems and 12 earthly branches with associated animal names (Table 1.9). Each year is identified by a heavenly stem and an terrestrial branch, in the order shown in Table 2.9. There are 60 year names, repeated consecutively. Each year in the cycle corresponds to a number from 1 to 60. The number 10 appears to be derived from an ancient weekly period, and the number 12 is related to the solar year.

The sexagenary cycle was originally used to count days. It came into use in the 13th century BCE during the Shang Dynasty. In the Han Dynasty (206 BCE-220 CE) the sexagenary cycle was used to number years, a practice that continues today. During the Tang Dynasty (618-907) the sexagenary cycle began to be used to number

Heavenly stems *(Gan)*		Earthly branches *(Zhi)*		
1	*Jia*	1	*Zi*	Rat
2	*Yi*	2	*Chou*	Ox
3	*Bing*	3	*Yin*	Tiger
4	*Ding*	4	*Mao*	Rabbit
5	*Wu*	5	*Chen*	Dragon
6	*Ji*	6	*Si*	Snake
7	*Geng*	7	*Wu*	Horse
8	*Xin*	8	*Wei*	Goat
9	*Ren*	9	*Shen*	Monkey
10	*Gui*	10	*You*	Chicken
		11	*Xu*	Dog
		12	*Hai*	Pig

Table 1.9.- **HEAVENLY STEMS AND EARTHLY BRANCHES**. The sexagesimal cycle is made up of 10 heavenly stems and 12 earthly branches. By combining them (as shown in the Table 2.9) we find 60 possible names for the years.

1	*Jia Zi*	21	*Jia Shen*	41	*Jia Chen*
2	*Yi Chou*	22	*Yi You*	42	*Yi Si*
3	*Bing Yin*	23	*Bing Xu*	43	*Bing Wu*
4	*Ding Mao*	24	*Ding Hai*	44	*Ding Wei*
5	*Wu Chen*	25	*Wu Zi*	45	*Wu Shen*
6	*Ji Si*	26	*Ji Chou*	46	*Ji You*
7	*Geng Wu*	27	*Geng Yin*	47	*Geng Xu*
8	*Xin Wei*	28	*Xin Mao*	48	*Xin Hai*
9	*Ren Shen*	29	*Ren Chen*	49	*Ren Zi*
10	*Gui You*	30	*Gui Si*	50	*Gui Chou*
11	*Jia Xu*	31	*Jia Wu*	51	*Jia Yin*
12	*Yi Hai*	32	*Yi Wei*	52	*Yi Mao*
13	*Bing Zi*	33	*Bing Shen*	53	*Bing Chen*
14	*Ding Chou*	34	*Ding You*	54	*Ding Si*
15	*Wu Yin*	35	*Wu Xu*	55	*Wu Wu*
16	*Ji Mao*	36	*Ji Hai*	56	*Ji Wei*
17	*Geng Chen*	37	*Geng Zi*	57	*Geng Shen*
18	*Xin Si*	38	*Xin Chou*	58	*Xin You*
19	*Ren Wu*	39	*Ren Yin*	59	*Ren Xu*
20	*Gui Wei*	40	*Gui Mao*	60	*Gui Hai*

Table 2.9.- **SEXAGESIMAL CYCLE**. The order in which the years occur in the sexagesimal cycle, each formed by a heavenly stem and a earthly branch. Each year is given a number from 1 to 60. The cycles occur consecutively.

months, a practice now in disuse.

The formulas we deduce below use modular arithmetic. The expressions will be of the form

$$y = 1 + \mathrm{mod}\left(\frac{x+a}{b}\right)$$

mod is the modulus or remainder operation of division, a and b are numerical parameters, and x and y are the independent and dependent variables. $\mathrm{mod}\left((x+a)/b\right)$ is an integer that cannot be greater than b; its possible values range from 0 to b - 1 . When x increases by one unit, y also increases by one unit. b is the cycle value (60 for the sexagenary or 10 for the heavenly stems). We determine the parameter a knowing y for a given value of x.

For reference, the lunar year *nian*, which began at 0 hours on February 2, 1984, of the Gregorian calendar and ended at 24 hours on February 19, 1985, corresponds to the name *Jia Zi*, or the first year of the sexagenary cycle.

Years	Sexagesimal cycle		Heavenly stems		Earthly branches		
2025	*Yi Si*	42	*Yi*	2	*Si*	6	Snake
2026	*Bing Wu*	43	*Bing*	3	*Wu*	7	Horse
2027	*Ding Wei*	44	*Ding*	4	*Wei*	8	Goat
2028	*Wu Shen*	45	*Wu*	5	*Shen*	9	Monkey
2029	*Ji You*	46	*Ji*	6	*You*	10	Chicken
2030	*Geng Xu*	47	*Geng*	7	*Xu*	11	Dog
2031	*Xin Hai*	48	*Xin*	8	*Hai*	12	Pig
2032	*Ren Zi*	49	*Ren*	9	*Zi*	1	Rat
2033	*Gui Chou*	50	*Gui*	10	*Chou*	2	Ox

Table 3.9.- **SEXAGESIMAL CYCLES**. The table shows the sexagesimal cycle for the years between 2025 and 2033, as well as the corresponding heavenly stems and earthly branches. We have found the results using formulas (1.9), (2.9) and (3.9).

The number of the year in the sexagenary cycle is calculated by

$$1+\mathrm{mod}\left(\frac{AD+56}{60}\right)=1+60\cdot\left[\frac{AD+56}{60}-\mathrm{int}\left(\frac{AD+56}{60}\right)\right] \tag{1.9}$$

AD is the Gregorian year to which the Chinese year belongs. Once the cycle number is known, we use Table 2.9 to determine the year's celestial stem and terrestrial branch.

Earthly branch and heavenly stem

With (1.9), we find the number of the sexagenary cycle corresponding to a year, which, as we have seen, combines a heavenly stem and a terrestrial branch. The number associated with the heavenly stem is calculated by

$$1+\mathrm{mod}\left(\frac{AD+6}{10}\right), \tag{2.9}$$

in 1984 the terrestrial stem was 1. The number associated with the earthly branch is

$$1+\mathrm{mod}\left(\frac{AD+8}{12}\right). \tag{4.9}$$

We determined the number 8 by imposing the condition that the earthly branch has the value 1 for the year 1984. In Table 2.9, we show the cycle number, the heavenly stem, and the terrestrial branch of the coming years.

Sexagenary cycle of days

We can use the sexagenary cycle to identify days. As a starting point, we take October 27, 2024, of the Gregorian calendar as the *Jia Zi* day, or the first day of the sexagenary cycle. The Julian Day at 12 hours of the previous day is 2,460,611.

To calculate the position of day *S* in the sexagenary cycle it is necessary to determine the Julian Day *JD,* which we find by identifying the day of the Chinese

calendar with the Gregorian day; then the number of the day in the cycle is

$$S = 1 + \mathrm{mod}\left(\frac{JD - 11}{60}\right)$$

we find the number 11 by imposing the condition that Julian Day 2,460,611 corresponds to $S = 1$. The number associated with the earthly branch is

$$T = 1 + \mathrm{mod}\left(\frac{S-1}{10}\right) = 1 + \mathrm{mod}\left(\frac{JD-1}{10}\right),$$

and the number of the heavenly stem is

$$H = 1 + \mathrm{mod}\left(\frac{S-1}{12}\right) = 1 + \mathrm{mod}\left(\frac{JD+1}{12}\right).$$

In Table 4.9, we give the sexagenary cycle of the first days of the lunar months of the year 2025 as an example.

Sexagenary cycle for the months

Towards the end of the 19th century, historians of Chinese astronomy associated a name or *jian* with the ordinary lunar months, identifying them with the names of the twelve terrestrial branches. An intercalary month has no *jian*. The first

Lunar months	Date first day of the month	Julian Day at 12 noon	S	T	H
1	2030/02/03	2 462 536	6	6	6
2	2030/03/04	2 462 565	35	5	11
3	2030/04/03	2 462 595	5	5	5
4	2030/05/02	2 462 624	34	4	10
5	2030/06/01	2 462 654	4	4	4
6	2030/07/01	2 462 684	34	4	10
7	2030/07/30	2 462 713	3	3	3
8	2030/08/29	2 462 743	33	3	9
9	2030/09/27	2 462 772	2	2	2
10	2030/10/27	2 462 802	32	2	8
11	2030/11/26	2 462 832	2	2	2
12	2030/12/25	2 462 861	31	1	7

Table 4.9.- **SEXAGESIMAL CYCLE FOR DAYS**. In the table, we calculate the heavenly stems, earthly branches, and sexagesimal cycle for the first days of the months of the year 2030. We determine the Julian Day of the Gregorian date using the algorithm developed in the Appendix III.

Heavenly stems of the year	Heavenly stems of the first month
Jia / Ji	*Bing*
Yi / Geng	*Wu*
Bing / Xin	*Geng*
Ding / Ren	*Ren*
Wu / Gui	*Jia*

Table 5.9.- **DETERMINATION OF THE SEXAGESIMAL CYCLE OF A MONTH**. We use the sexagesimal cycle to identify months, following the same procedure for naming years. The table allows us to determine the sexagesimal cycle of a month. The first column is the heavenly stem of the year, and the second column is the heavenly stem of the year's first month. The heavenly stem begins with *Yin*, so the first month of *nian* is identified as *Yin*. If the year is *Jia* or *Ji*, then the first month of that year is *Bing Yin*. If, for example, the heavenly stem of the year is *Ding*, then the first month is *Ding Yin*. Once the name of the year's first month is known, we determine the name of any other month of the year.

month, or *Zi* month, is the 11th, containing the winter solstice; the 12th month is *Chou*, and the first month of the lunisolar year, *nian*, is *yin*.

In the past, the sexagenary cycle was used to identify the months. The earthly branch is the *jian* of the month, such that the first month of the year *nian* always has the *jian Yin*, the second month is *Mao*, the third *Chen* and so on.

There are 60 months in 5 years, a complete sexagenarycycle; this allows us to relate the heavenly stem of the year to the heavenly stem of the year's first month. The starting point is that if the year is *Jia*, the celestial stem of the first month of that year is *Bing,* which corresponds to the name *Bing Yin* of the sexagenary cycle. The year will have the heavenly stem *Ji* five years later, but since 60 months have passed, the first month of that year will again be *Bing Yin*. Therefore, the heavenly stems of the *Jia/Ji* years correspond to a first month of celestial stem *Bing*.

The year following a *Jia* year will have a *Yi* heavenly stem; during that year, twelve months will have passed, and the first month of the new year is *Wu Yin*. Thus, the heavenly stem of the *Yi* year is associated with the heavenly stem of the first month, *Wu*. Five years later, the celestial stem is *Geng*, which will also have the *Wu* heavenly stem as its first month. In other words, *Yi/Geng* heavenly stems correspond to a *Wu* heavenly stem in the first month.

Thus, we can follow the reasoning, and the result is Table 5.9, which allows us to determine the sexagenary cycle of any month.

For example, we find the sexagenary denomination of the sixth month of 2025. Using formula (1.9), we find that this year has the number 42, corresponding to *Yi Si*. Since the heavenly stem is *Yi*, Table 5.1 shows that the first month of that year is *Wu Yin*, and the sixth month will be *Wu Wein*.

In addition to the lunar months, whose beginning coincides with the new Moon, we distinguish (see below) the solar months, which are regulated by the minor solar terms. If the minor solar terms regulate them, we will call them *jieqì* solar months

and reserve the name *zhongqì* solar months or simply solar months for those regulated by the major solar terms.

The *jieqì* solar months begin at 0 hours when the minor solar term occurs. The months are numbered with the terrestrial branches. The *Zi* or first month of the count, begins on the day the J11 term occurs, the second-month *Chou* begins with J12 and so on. As we will see later, the solar months, whether *jieqi* or *zhongqi*, have 29, 30, 31 or 32 days, depending on the time of year.

The chinese era

In China, there is no continuous counting of years; the era is not part of the tradition of the Chinese calendar. The years began to be numbered with the new emperor, chronological dating being completed with the sexagenary cycle. For various reasons, there were numerous alterations in the years of the emperor, which makes it difficult to fix a date in the past.

In order to provide the Chinese calendar with an era, the beginning of the counting of years was set at 2637 BC (or -2636 in astronomical notation) when the calendar was implemented, corresponding to the 61st year of Emperor Huangdi. Other authors prefer to set the date of origin of the Chinese era as the year in which this emperor began to reign, which occurred 60 years earlier, that is, in 2697 BC (or -2696). Therefore, 2000 CE corresponds to the Chinese year 4636 or 4696 according to the adopted criterion.

	Wood - 1	Fire - 2	Earth - 3	Metal - 4	Water - 5
Rat -1	1	13	25	37	49
Ox - 2	2	14	26	38	50
Tiger - 3	51	3	15	27	39
Rabbit - 4	52	4	16	28	40
Dragon - 5	41	53	5	17	29
Snake - 6	42	54	6	18	30
Horse - 7	31	43	55	7	19
Goat - 8	32	44	56	8	10
Monkey - 9	21	33	45	57	9
Chicken - 10	22	34	46	58	10
Dog - 11	11	23	35	47	59
Pig - 12	12	24	36	48	60

Table 6.9.- **THE SEXAGENARY CYCLE WITH ELEMENTS AND ANIMALS.** The table shows the number of the sexagenary cycle using the combination of elements and animals. The first year of the cycle corresponds to Wood-Rat, which is associated with *Jia-Zi.*

By choosing one modality or the other, we obtain a coincidence between the sexagenary cycles of the two types of eras since they differ in 60 years, that is, a complete cycle. For example, 2025 is in cycle 77 of the first era and in position 42 or *Jia Si*, and the same position in the cycle would be if we used the second era, except that in this case, we would be in cycle 78.

The sexagenary cycle with the five elements

Another way to apply the sexagenary cycle is to combine the five elements —wood (*mu*), fire (*hou*), earth (*tu*), metal (*jin*), and water (*shui*)— with the twelve animals associated with the earthly branches. The sequence of the sexagenary cycle is illustrated in Table 6.9.

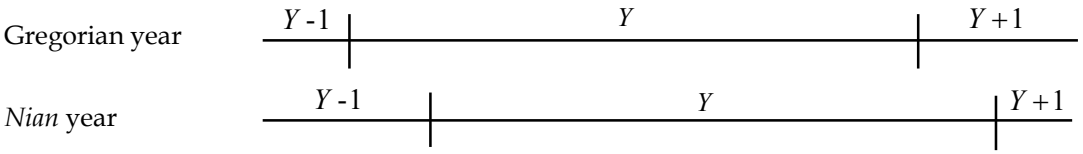
We associate the number 1 with wood, 2 with fire, and so on, which we identify with the letter *e*. We also give numbers to animals: the rat corresponds to 1, the ox to 2, and so on, generically giving it the letter *a*.

We identify each year with the numbers (*e, a, k*), where *k* is the number of the sexagenary cycle. The cycle that began in 2697 BC, the first year of Emperor Huang Di, is given the value 1, the year to which the combination (1,1,1) corresponds.

The correspondence with the sexagenary cycle constructed with the earthly branches and the heavenly stems is such that *Ji Zi* corresponds to Wood-Rat.

10.- Determining the Chinese calendar

This chapter will explain the techniques used to find the Chinese calendar for a given year. As we have already indicated, the Chinese year *nian* runs through two Gregorian years. As a convention, we associate the *nian* year with the Gregorian year whose first day, January 1, is closest to the Chinese New Year's day. Thus, the *nian* year Y runs between 20 and 51 days * in the Gregorian year $Y+1$ and the rest of the days in the Gregorian year Y.



Ilustration 1.10.- **GREGORIAN YEAR AND *NIAN* YEAR**. The illustration shows how the *nian* year Y is located between two Gregorian years, Y and $Y+1$. Between the beginning of the Gregorian year Y and the beginning of the *nian* year Y there can be between 20 and 51 days; that is, the *nian* year can begin between January 21 and February 21.

We associate the *sui* year with the Gregorian year Y, whose first day, January 1st, is closest to the first day of the month following the 11th **. Sometimes, the *sui* year Y begins in the Gregorian year $Y-1$, but other times, it occurs in the year Y. The *sui* year Y sometimes ends in the Gregorian year Y and other times in the year $Y+1$.

To determine the Chinese calendar for the year *nian* Y it is necessary to analyze the previous year, i.e. $Y-1$, and the following year $Y+1$ ***. As always, the results are referred to the time zone of the meridian at 120º east.

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* The earliest Chinese New Year falls on January 21, and the latest on February 21. In the first case, 20 days of the Gregorian year $Y+1$ belong to the Chinese year Y, while in the second case, 51 days (31 days of January and 20 of February) belong to the Gregorian year $Y+1$.

** The month following the 11th is not always the 12th. Exceptionally, it can be the intercalary-11.

*** We have to do the calculations for year $Y+1$ because it is possible that the intercalary month of that year is after month 11 or month 12 of the year Y.

1. **Finding the Gregorian dates of solar terms**

* From astronomical theory, we find the Julian Day of the solar terms of the years Y -1, Y, and $Y+1$, which we will call $t(n)$, where n is a number from 1 to 25. Thus, $t(1)$, $t(13)$, and $t(25)$ are the winter solstices of the years Y -1, Y, and $Y+1$.
* From $t(n)$, we find $td(n)$, which is the Gregorian date of the day on which the solar term occurs, corresponding to the Julian Day at 0 hours of the day on which the solar term occurs.
* To determine $td(n)$ from $t(n)$ we previously calculated the associated Julian day $JD(n)$ and we take into account

$$\text{if } t(n)-\text{int}\left[t(n)\right]<0.5 \text{ then } JD(n)=\text{int}\left[t(n)\right]-0.5$$
$$\text{if } t(n)-\text{int}\left[t(n)\right]\geq 0.5 \text{ then } JD(n)=\text{int}\left[t(n)\right]+0.5$$

and from the Julian day $JD(n)$ we find the Gregorian date $td(n)$ (Appendix III).

2. **Finding the Gregorian dates of the new Moons**

* From astronomical theory we find the Julian Day of the new Moons $l(m)$ of the years Y -1, Y and $Y+1$, where $m=1$ corresponds to the new Moon that arrives immediately before or on the same day as the winter solstice of the year Y -1, that is, it is the Moon with which the 11th month of the year Y -1 begins; and the last value of m corresponds to that of the new Moon on which the 11th month of the year $Y+1$ begins.
* From $l(m)$, we find $ld(m)$, which is the Julian Day of the Gregorian date of the day on which the new Moon occurs. That is the date that corresponds to Julian Day at 0 hours of the new Moon's day and is the lunar month's first day.

3. **Associate solar terms with lunar months**

* We represent in a table the dates of the beginning of each month, which correspond to the dates of the new Moons $ld(m)$, together with the dates of the solar terms $td(n)$, associating each solar term with the month in which they occur.

Table 1.10.- **CALENDAR FOR THE YEAR 2028.** The first column numbers the new Moons, starting with the one that begins in the 11th month of the year 2027, the month that contains the winter solstice of the year 2027. The second column is the Gregorian date of the new Moon or first day of the lunar month. The third column numbers the solar terms, starting with the winter solstice 2027. Finally, the fifth column is the designation of the solar terms, with Z11 corresponding to the winter solstice. Note that every year has 12 solar terms that are numbered consecutively.
The dates in bold correspond to the winter solstices and the new Moons of the months they occur.
The *sui* year runs from the month after the 11th month of the year 2027 (or 2028) to the month after the 11th month of the year 2028 (or 2029), which does not always have solar term Z12, since exceptionally, the month after the 11th is the month intercalary-11.
We can see that the *sui* year 2028 is intercalary because it has 13 months. The intercalary month is the one numbered $m=8$ because it has no associated solar term.
In both years under review, the first month of the *nian* year is the second month after the 11th month, i.e. the month with solar term Z1. In exceptional cases, there may be intercalary months 11 and 12, where the new year will be the third month after the 11th.

m	News Moons *ld*(*m*)	*n*	Solar terms *td*(*n*)	*Z*	Notes
1	**2027/11/28**	1	**2027/12/22**	Z11	Winter Solstice 2027. Month 11
2	2027/12/28	2	2028/01/20	Z12	Beginning of the year *sui* 2028
3	2028/01/26	3	2028/02/19	Z1	New Year 2028. Beginning of the *nian* year 2028
4	2028/02/25	4	2028/03/20	Z2	Spring Equinox 2028
5	2028/03/26	5	2028/04/19	Z3	
6	2028/04/25	6	2028/05/20	Z4	
7	2028/05/24	7	2028/06/21	Z5	Summer Solstice 2028
8	2028/06/23		---	---	Intercalary month
9	2028/07/22	8	2028/07/22	Z6	
10	2028/08/20	9	2028/08/22	Z7	
11	2028/09/19	10	2028/09/22	Z8	Autumn equinox 2028
12	2028/10/18	11	2028/10/23	Z9	
13	2028/11/16	12	2028/11/22	Z10	
14	**2028/12/16**	13	**2028/12/21**	Z11	Winter Solstice 2028. Month 11
15	2029/01/15	14	2029/01/20	Z12	Beginning of the year *sui* 2029
16	2029/02/13	15	2029/02/18	Z1	New Year 2029. Beginning of the *nian* year 2029
17	2029/03/15	16	2029/03/20	Z2	Spring Equinox 2029
18	2029/04/14	17	2029/04/20	Z3	
19	2029/05/13	18	2029/05/21	Z4	
20	2029/06/12	19	2028/06/21	Z5	Solsticio de verano 2029
21	2029/07/11	20	2029/07/22	Z6	
22	2029/08/10	21	2029/08/23	Z7	
23	2029/09/08	22	2029/09/23	Z8	Summer Solstice 2029
24	2029/10/08	23	2029/10/23	Z9	
25	2029/11/06	24	2029/11/22	Z10	
26	**2029/12/05**	25	**2029/12/21**	Z11	Winter Solstice 2029. Month 11

4. **Identify the 11 months of the years *Y*-1, *Y* and *Y*+1**
* We identify the 11th months of the years $Y-1$, Y and $Y+1$, which are those where the winter solstices occur.

5. **Find out if *sui* years *Y* and *Y*+1 are embolismic**
* We calculate the lunar months in the *sui* year *Y*, that is, the number of months from the end of the 11th month of the year $Y+1$ to the end of the 11th month of the year *Y*; and likewise for the year $Y+1$, that is, the months elapsed from the end of the 11th month of the year *Y* to the end of the 11th month of the year $Y+1$. If there are 12 months, they will be common years; if there are 13 months, they will be embolismic years.

6. **Find the intercalary month of the years *Y* or *Y*+1**
* From the table of new Moons and solar terms, we find out the intercalary lunar month of the years *Y* or *Y*+1 in the case of embolismic years, understanding that the intercalary month does not have a solar term.

7. **Find out the New Year's day of the year *nian Y***
* The first month of the *nian* year *Y* will be the second month after the 11th month of the year *Y*-1, without considering any intercalary month that may exist, and New Year's day will be the first day of the first month of the year.

8. **Numbering of the months of the year *nian Y***
* Finally, we number the lunar months of the year Y starting with month 1 and naming the intercalary month, if it exists, with the same number as the previous month.

9. **Calendar for the year 2028**
* In Table 1.10, we show the procedure for determining the calendar for the year 2028. From astronomical theory, we find the dates of the new Moons from the end of the Gregorian year *Y*-1 to the end of the Gregorian year *Y*+1. Also, we find the solar terms from astronomical theory, which we associate with the corresponding new Moons.

Table 2.10.- **CALENDAR FOR THE YEARS 2028 AND 2029**. On the next page, we show the simplified calendar for 2028 and 2029, taking the data from Table 1.10. In the first column is the name of the months. The first month is the second after the month that contains the winter solstice (not counting the intercalary months, if they exist). In the two years of this table, the first month is the second after the month that has the winter solstice. The intercalary month is the one that comes after the 5th month and is therefore called intercalary-5. The year 2029 is common, with 12 lunar months, where all months have a solar term.
In the tables, we also represent the duration of each lunar month, the day of the week on which the month begins, the date of the solar term, and the name of this term, taking into account that the month Z11 has the winter solstice.
The first month is usually called Z1, although exceptionally, it may happen that this month does not have a solar term and is not intercalary, in which case Z1 would be the second month of the *nian* year.

Number of the month	Beginning of the month	Duration month	Day of the week	Date solar term	Solar term
		2028			
1	2028/01/26	30	Wed.	2028/02/19	Z1
2	2028/02/25	30	Fri.	2028/03/20	Z2
3	2028/03/26	30	Sun.	2028/04/19	Z3
4	2028/04/25	29	Tue.	2028/05/20	Z4
5	2028/05/24	30	Wed.	2028/06/21	Z5
5i	2028/06/23	29	Fri.	---	---
6	2028/07/22	29	Sat.	2028/07/22	Z6
7	2028/08/20	30	Sun.	2028/08/22	Z7
8	2028/09/19	29	Tue.	2028/09/22	Z8
9	2028/10/18	29	Wed.	2028/10/23	Z9
10	2028/11/16	30	Thu.	2028/11/22	Z10
11	2028/12/16	30	Sat.	2028/12/21	Z11
12	2029/01/15	29	Mon.	2029/01/20	Z12
		2029			
1	2029/02/13	30	Tue.	2029/02/18	Z1
2	2029/03/15	30	Thu.	2029/03/20	Z2
3	2029/04/14	29	Sat.	2029/04/20	Z3
4	2029/05/13	30	Sun.	2029/05/21	Z4
5	2029/06/12	29	Tue.	2028/06/21	Z5
6	2029/07/11	30	Wed.	2029/07/22	Z6
7	2029/08/10	29	Fri.	2029/08/23	Z7
8	2029/09/08	30	Sat.	2029/09/23	Z8
9	2029/10/08	29	Mon.	2029/10/23	Z9
10	2029/11/06	29	Tue.	2029/11/22	Z10
11	2029/12/05	30	Wed.	2029/12/21	Z11
12	2030/01/04	30	Fri.	2030/01/20	Z12

m	New Moons *ld*(*m*)	*n*	Solar terms *td*(*n*)	*Z*	Notes
1	**1812/12/04**	1	**1812/12/22**	Z11	Winter Solstice 1812. Month 11
2	1813/01/03	2	1813/01/20	Z12	Beginning of the year *sui* 1813
3	1813/02/01	3	1813/02/19	Z1	New Year 1813. Beginning of the *nian* year 1813
4	1813/03/03	4	1813/03/21	Z2	Spring Equinox 1813
5	1813/04/01	5	1813/04/20	Z3	
6	1813/05/01	6	1813/05/21	Z4	
7	1813/05/30	7	1813/06/22	Z5	Summer Solstice 1813
8	1813/06/28	8	1813/07/23	Z6	
9	1813/07/27	9	1813/08/23	Z7	
10	1813/08/26	10	1813/09/23	Z8	Autumn equinox 1813
11	1813/09/24	---	---	---	Empty or fake month
12	1813/10/24	11	1813/10/24	Z9	
---	---	12	1813/11/22	Z10	Double month
13	1813/11/23	13	1813/12/22	Z11	Winter Solstice 1813. Month 11
14	1813/12/23		---	...	Intercalary month
15	1814/01/21	14	1814/01/23	Z12	Beginning of the year *sui* 1814
---	---	15	1814/02/19	Z1	Double month
16	1814/02/20	16	1814/03/21	Z2	New Year 1813. Beginning of the *nian* year1814
17	1814/03/22	---	---	---	Empty or fake month
18	1814/04/20	17	1814/04/21	Z3	
19	1814/05/20	18	1814/05/22	Z4	
20	1814/06/18	19	1814/06/22	Z5	Summer Solstice 1814
21	1814/07/17	20	1814/07/23	Z6	
22	1814/08/15	21	1814/08/24	Z7	
23	1814/09/14	22	1814/09/24	Z8	Autumn equinox 1814
24	1814/10/13	23	1814/10/24	Z9	
25	1814/11/12	24	1814/11/23	Z10	
26	1814/12/12	25	1814/12/22	Z11	Winter Solstice 1814. Month 11

Number of the month	Beginning of the month	Duration month	Day of the week	Date solar term	Solar term
		1813			
1	1813/02/01	30	Mon.	1813/02/19	Z1
2	1813/03/03	20	Wed.	1813/03/21	Z2
3	1813/04/01	30	Thu.	1813/04/20	Z3
4	1813/05/01	29	Sat.	1813/05/21	Z4
5	1813/05/30	29	Sun.	1813/06/22	Z5
6	1813/06/28	29	Mon.	1813/07/23	Z6
7	1813/07/27	30	Tue.	1813/08/23	Z7
8	1813/08/26	29	Thu.	1813/09/23	Z8
9	1813/09/24	30	Fri.	----	----
10	1813/10/24	30	Sat.	1813/10/24	Z9
---	---	---	---	1813/11/22	Z10
11	1813/11/23	30	Tue.	1813/12/22	Z11
11i	1813/12/23	29	Thu.	----	----
12	1814/01/21	30	Fri.	1814/01/23	Z12
---	---	---	---	1814/02/19	Z1
		1814			
1	1814/02/20	30	Sun.	1814/03/21	Z2
2	1814/03/22	29	Tue.	---	---
3	1814/04/20	30	Wed.	1814/04/21	Z3
4	1814/05/20	29	Fri.	1814/05/22	Z4
5	1814/06/18	29	Sat.	1814/06/22	Z5
6	1814/07/17	29	Sun.	1814/07/23	Z6
7	1814/08/15	30	Mon.	1814/08/24	Z7
8	1814/09/14	29	Wed.	1814/09/24	Z8
9	1814/10/13	30	Thu.	1814/10/24	Z9
10	1814/11/12	30	Sat.	1814/11/23	Z10
11	1814/12/12	29	Mon.	1814/12/22	Z11
12	1815/01/10	30	Tue.	1815/01/21	Z12

Table 3.10.- **CALENDAR FOR THE YEARS 1813 AND 1814**. The calendar for 1813/1814 was exceptional. There were two empty months, two double months and one intercalary month. When this situation occurs, we encounter the problem of discovering the intercalary month. We examine the *sui* years 1813 and 1814. The *sui* year 1813 begins in the next lunar month in which the winter solstice falls. From the table, we find that it has 12 lunar months; therefore, it is not an intercalary year, so in this period, there is no intercalary month, so lunar month 9 is a empty or fake month.
The *sui* year 1814 has 13 lunar months, so it is intercalary. The intercalary month is the first empty month of the *sui* period, so the intercalary month is the 11i. The remarkable thing about this calendar is that after the intercalary month, a double month appears again and then, to compensate, an empty month comes along, which is also fake.
Note that the year *sui* 1813 is common, but the year *nian* 1813 is intercalary, and the year *sui* 1814 is intercalary, but the *nian* 1814 is common.

* In Table 1.10, we determine the beginning of the *sui* year, which is the lunar month that comes immediately after the winter solstice. The *sui* year 2028 has 13 lunar months; therefore, it is an intercalary year. The table shows that the lunar month that begins on June 23, 2028, does not have a solar term. Therefore, that month will be the intercalary month. The *sui* year 2029 begins with the new Moon on January 15, 2029, which is 12 months; therefore, it is a common year.
* In Table 2.10, we represent the simplified calendar for the years 2028 and 2029, indicating the duration of the lunar months, the weekly day on which each month begins, and the names of the solar terms that correspond to each lunar month.
* As an example of a singular year, we represent the calendar for 1813/1814 in Table 3.10. There are three empty months, i.e. lunar months that do not have major solar terms. When this situation occurs, it is necessary to determine which of these months is the intercalary month. The procedure is to determine the lunar months of the *sui* years 1813 and 1814. We find that they are 12 and 13, so the *sui* intercalary year is 1814. The intercalary month is the first empty month of the *sui* year 1814, i.e. the lunar month that begins on December 23, 1813. Although this month belongs to the *sui* year 1813, it nevertheless belongs to the *nian* year 1814.

11.- Chinese solar calendar

We classify the Chinese calendar as lunisolar because the Sun's motion determines which years have 13 months, similar to how the Jewish or Christian calendars are regulated. However, the Chinese calendar follows the Sun throughout its apparent motion around the Earth, not limiting itself to a specific milestone (an equinox or a solstice) to regulate the lunisolar calendar. This allows it to not only know when a year is embolismic but also identify which is the intercalary month, which does not occur in other lunisolar calendars.

Therefore, the Chinese calendar combines a lunar calendar (the month begins with the new Moon) and a solar calendar (defined by the Sun's apparent movement). Combining these two calendars, we find the Chinese lunisolar calendar, the *nian* calendar.

Requiring that the winter solstice occurs in the 11th lunar month allows us to know if the year is common or embolismic: if the *sui* year (the one that begins with the month after the 11th) has 13 lunar months, it is embolismic, and if it has 12 months, it is common.

The spring equinox is the calendar-regulating milestone in the Jewish and Christian calendars. However, in these calendars, the intercalary month is chosen arbitrarily; this is where the uniqueness of the Chinese calendar lies.

Solar month

We divide the Sun's ecliptic orbit into 12 parts of 30 degrees of longitude each, which gives us 12 solar months or the time elapsed for the Sun to travel through each of these parts. We have found that the Sun's ecliptic velocity depends on its position in its elliptical orbit, being maximum in the perihelion zone and minimum in the aphelion zone; this means that the length of the solar months will depend on the time of year.

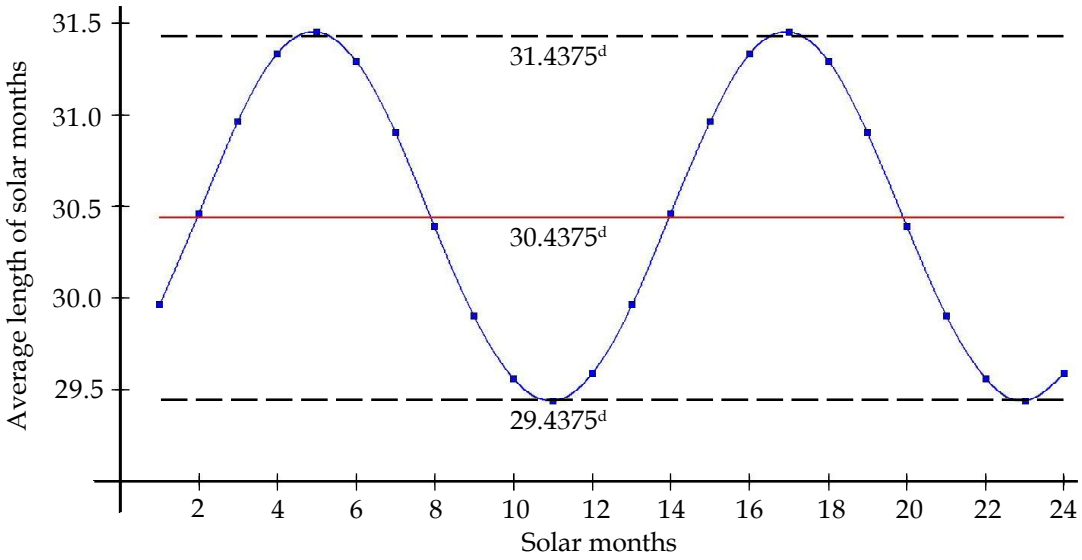
We define the astronomical solar month in two different ways. The astronomical solar month *jieqi* is the period between two consecutive minor solar terms. The astronomical solar month *zhongqi* is the time elapsed between two major solar terms.

For calendar purposes, we are interested in the solar month *zhongqi* (or calendar solar month or simply solar month) which begins at 0 hours on the day the corresponding solar term occurs; that is, it has an integer number of days.

We define the first solar month as the one containing the first major solar term Z1, that is, the one that begins on the day when the Sun's apparent ecliptic longitude is 330^{o}. We will identify the solar months in the same way as the major solar terms, with the letter Z followed by a number from 1 to 12.

Solar month	Years			Orbital velocity (º/day)
	1000	2000	3000	
Z1	30.23	29.96	29.74	1.001
Z2	30.77	30.46	30.17	0.985
Z3	31.22	30.96	30.68	0.969
Z4	31.46	31.33	31.12	0.957
Z5	31.42	31.45	31.39	0.954
Z6	31.11	31.29	31.39	0.959
Z7	30.62	30.90	31.13	0.971
Z8	30.09	30.39	30.68	0.987
Z9	29.66	29.90	30.18	1.003
Z10	29.44	29.56	29.75	1.015
Z11	29.47	29.44	29.50	1.019
Z12	29.76	29.59	29.50	1.014

Table 1.11.- **AVERAGE DURATION OF ASTRONOMICAL SOLAR MONTHS**. The astronomical solar month is the time between two consecutive major solar terms. In the first column of the table, the major solar term that appears is the beginning of the solar month, which is used to name it. The months' duration is expressed in days and refers to average times expressed in TT (but as it is a difference in time, it coincides with UTC). Although the Sun describes the same angle of the ecliptic in each astronomical solar month, it takes a different time to make the journey depending on the position of the Sun in the ecliptic.
When the Sun moves closer to the perihelion (winter solar months: Z9, Z10, Z11 or Z12) it takes less time, because according to Kepler's second law the velocity of the Sun is greater. However, in the solar months located in the summer (Z3, Z4, Z5 or Z6) the duration is longer because the Sun is close to the aphelion and has less velocity. This variation is reflected in the Chinese calendar, as it is very unlikely that the intercalary months are in winter and much more likely in summer.
The table shows that the length of the astronomical solar months varies over time due to the precession of the Earth's perihelion. The length of some months increases while that of others decreases. The length of months Z12, Z1, Z2, Z3, and Z4 decreases; the length of months Z6, Z7, Z8, Z9, and Z10 increases, and the length of months Z5 and Z11 remains constant.
The longest average duration in the year 2000 is the month beginning with solar term Z5, which has a mean duration of 31.45 days, and the shortest duration is the month beginning with Z11, 29.44 days. This is a difference of just over two days, which may be greater if we calculate the true durations and not the averages.
In the last column is the average orbital velocity during each solar month expressed in degrees per day for the year 2000.
We have obtained the results by exclusively considering the Sun's mean longitude corrected by the centre equation, and considering the variation of the orbital elements.
Each solar month can only have two lengths. For example, if the average length of month Z10 is 29.59 days, then the calendar solar month can have either 29 or 30 days.



Ilustration 1.11.- **AVERAGE DURATION OF ASTRONOMICAL SOLAR MONTHS**. We represent Table 1.11 graphically, showing the variation in the average length of the astronomical solar months. The horizontal axis indicates the months starting from the year 2000. The continuous horizontal straight line represents the mean value of the solar month (30.4375 days), and the vertical axis is the average length of the months. The dispersion of the extreme values is approximately 1 day. The dashed lines correspond to one day more or one day less than the average value of the solar month. The maximum value of the length of the astronomical month corresponds to Z5, and the lowest value is Z11.

Table 1.11 shows the average length of the calendar solar months for the years 1000, 2000 and 3000. Since the Sun describes the 360^{o} of its orbit in approximately 365.25 days, the solar month will have a duration of 30.4375 on average. Ilustration 1.11 shows the dispersion of the average values for the duration of the astronomical solar months. The horizontal line in Ilustration 1.11 is the average length of the solar months, that is, their length if all the months had the same length. The extreme dispersions with respect to the average value are approximately one day, so all the durations are practically between the two discontinuous horizontal lines, which correspond to the durations 30.4375+1 and 30.4375-1.

Since the Sun's motion is very regular, the true durations of the astronomical solar months have little dispersion with respect to the mean values in Table 1.11. In Table 2.11, we give as an example the durations of the odd-numbered astronomical solar months for several years, and we see that the dispersion from one year to another is minimal with respect to the result in Table 1.11.

Given the above circumstance, each solar month can only have two possible durations. For example, the mean value of the astronomical month of the fourth month is 31.33 days, then the calendar solar month associated with Z4 may sometimes have 31 days and sometimes 32 days, depending on the moment when the major solar term Z4 occurs (see Illustration 2.11). Indeed, if the solar term Z4 begins later than 16.08 hours $(= 24 - 24 \times 0.33)$, then the month will have 32 days; but if the solar term begins before 16.08 hours, then the month will have 31 days.

In Table 3.12, we show the possible durations of the solar months of the

Month	Year					
	2025	2026	2027	2028	2029	2030
Z1	29.955	29.954	29.952	29.952	29.954	29.953
Z3	30.957	30.957	30.959	30.959	30.958	30.957
Z5	31,449	31.450	31.454	31.453	31.454	31.454
Z7	30.907	30.907	30.908	30.906	30.907	30.910
Z9	29.906	29.907	29.905	29.903	29.904	29.906
Z11	29.446	29.444	29.444	20.445	29.445	29.443

Table 2.11.- **TRUE DURATION OF ASTRONOMICAL SOLAR MONTHS**. We see in the table that the variation between the true durations of the astronomical solar months from one year to the next is small compared to the mean values in Table 1.11. In the table we calculate the durations of the odd months.

Solar month	Average duration (year 2000)	%			
		29 days	30 days	31 days	32 days
Z1	29.96	4	96	0	0
Z2	30.46	0	54	46	0
Z3	30.96	0	4	96	0
Z4	31.33	0	0	67	33
Z5	31.45	0	0	55	45
Z6	31.29	0	0	71	29
Z7	30.90	0	10	90	0
Z8	30.39	0	61	39	0
Z9	29.90	10	90	0	0
Z10	29.56	44	56	0	0
Z11	29.44	56	44	0	0
Z12	29.59	41	59	0	0

Table 3.11.- **FREQUENCY OF THE DURATION OF THE SOLAR MONTHS OF THE CALENDAR**. For the average value of the astronomical solar months for the year 2000, we have found the frequency of the durations of each month. We have found that each month can only have two different durations. There are months of 29, 30, 31 and 32 days. Only the months Z4, Z5 and Z6 can have months of 32 days' duration since, during these months, the Sun is close to the aphelion and, therefore, has a lower orbital velocity, taking longer to travel the 30 degrees of ecliptic longitude that corresponds to each month. The months from Z9 to Z1 can have months of 29 days' duration because they occur when the Sun is close to the perihelion, where the orbital velocity is faster, and it takes less time to travel the 30 degrees of ecliptic longitude that each month has.

Solar month	Years												Average value
	2025	2026	2027	2028	2029	2030	2031	2032	2033	2034	2035	2036	
Z1	30	30	30	30	30	30	30	30	30	30	30	30	30.00
Z2	31	31	30	30	31	31	30	30	31	31	30	30	30.55
Z3	31	31	31	31	31	31	31	31	31	31	31	31	31.00
Z4	31	31	31	32	31	31	31	32	31	31	31	32	31.25
Z5	31	32	32	31	31	32	32	31	31	32	32	31	31.50
Z6	32	31	31	31	32	31	31	31	32	31	31	31	31.25
Z7	31	31	31	31	31	31	31	31	31	31	31	31	31.00
Z8	30	30	30	31	30	30	30	31	30	30	30	31	30.25
Z9	30	30	30	30	30	30	30	30	30	30	30	30	30.00
Z10	29	30	30	29	29	30	30	29	29	30	30	29	29.50
Z11	30	29	29	30	30	29	29	30	30	29	29	30	29.50
Z12	29	30	30	29	29	30	30	29	29	30	30	29	29.50
Total	365	366	365	365	365	366	365	365	365	366	365	365	365.30

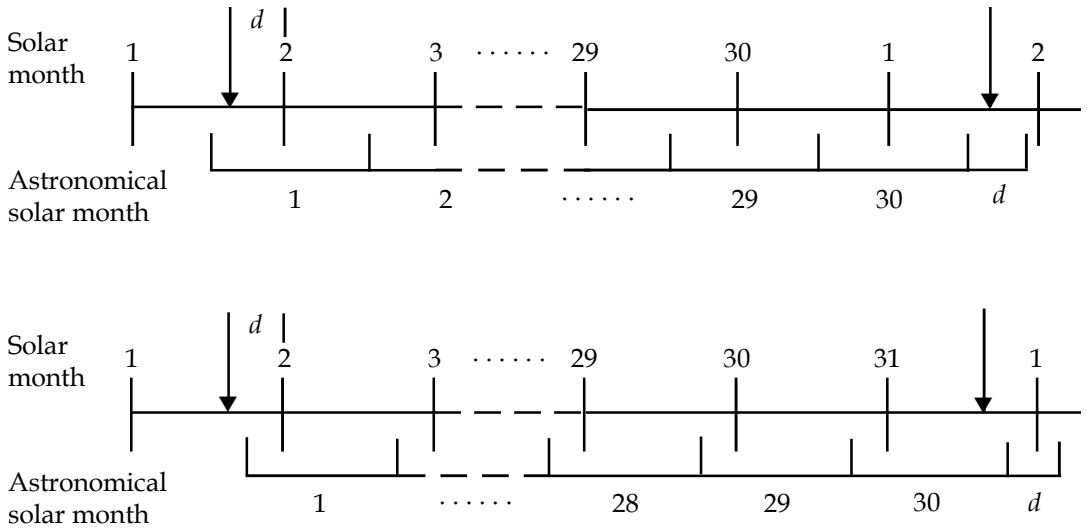
Table 4.11.- **DURATION OF SOLAR MONTHS**. The table shows the solar months for the years 2025-2036. We first notice that their duration is variable and can have two possible lengths. The duration of the solar year understood as the time between two consecutive Z1 solar months, is 365 or 366 days. We note that the sequence of the years is that of the Gregorian calendar, three years of 365 days followed by a leap year of 366 days; however, there is no coincidence between the leap years of the Gregorian calendar (those divisible by 4) and those of the Chinese solar calendar.
The last column contains the average values of the duration of the solar months during the period considered, which are very close to the average values in Table 2.11.

calendar, with an indication of the frequency with which they occur. The frequency will change slightly for other years, but the possible durations of the months will remain the same for several thousand years. To calculate them, we follow the following process, where we calculate the frequencies of the month Z1, which has an average duration of 29.96 days (Table 2.11). Let x and y be the number of times that the month Z1 has 29 and 30 days in a period, which we will assume to be 100 years

$$29.96 = \frac{29x + 30y}{100}; \quad x + y = 100 \quad \Rightarrow \quad x = 4; \quad y = 96.$$

In Table 4.11, we show the duration of solar months for several years, verifying that each can only have two different durations. In the same table, we find that the average value for the period considered is very close to the average value in Table 1.11.

In Table 4.11, we also find that solar years, understood as the time periods that begin with the day on which solar term Z1 occurs, can be 365 or 366 days long,



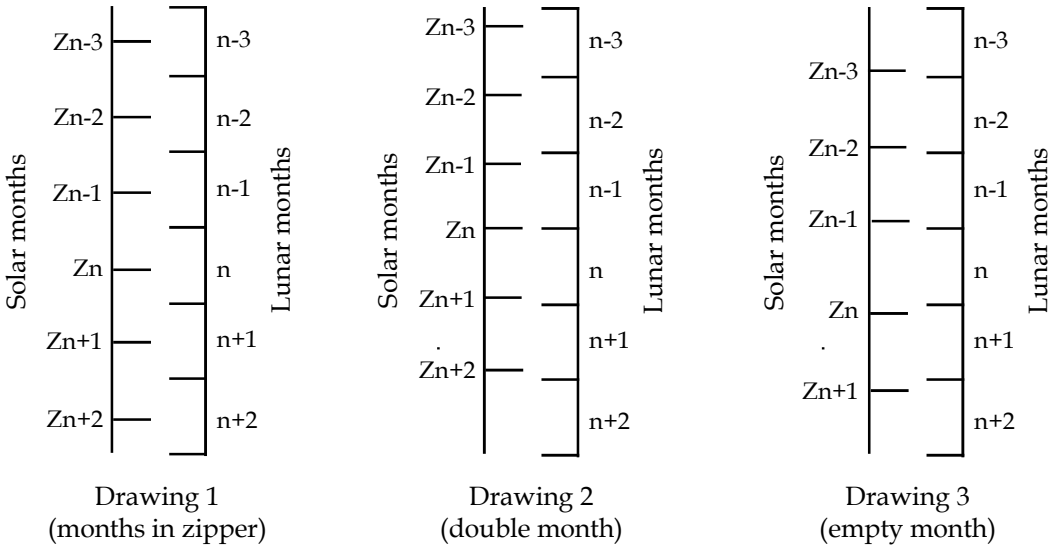
Ilustration 2.11.- **THE TWO POSSIBLE DURATIONS OF SOLAR MONTHS**. The diagram schematically explains the two possible durations of solar months. We use the example of an astronomical solar month that lasts 30+*d* days. The possible durations of the corresponding calendar solar month can be 30 and 31 days, depending on the time of day when the associated major solar term occurs. The calendar solar month begins at 0 hours of the day when the corresponding major solar term occurs.
In the drawing, we show a vertical arrow marking the limit, which is *d* days away from the beginning of the second day of the solar month. In the first drawing, we find the case where the solar term arrives before the mark; from the moment of the solar term to the beginning of the next day there are more than *d* days. The end of the solar month, which lasts 30+*d* days, occurs in the month following the 30th of the solar month, so this day corresponds to the 1st of the following solar month since the new solar term occurs in that month.
In the second drawing, we show the case in which the solar term occurs less than *d* days from day 2 of the solar month; then the lunation ends on the following day of the 31st of the solar month, this is the last day of the month and the following is month 1, the one in which the solar term occurs is the first day of the new month.

just like the year of the Gregorian calendar. However, we note that although one of them is a leap year of 366 days every four years, the leap years of the Chinese solar calendar do not coincide with the leap years of the Gregorian calendar.

Saolar and lunar months

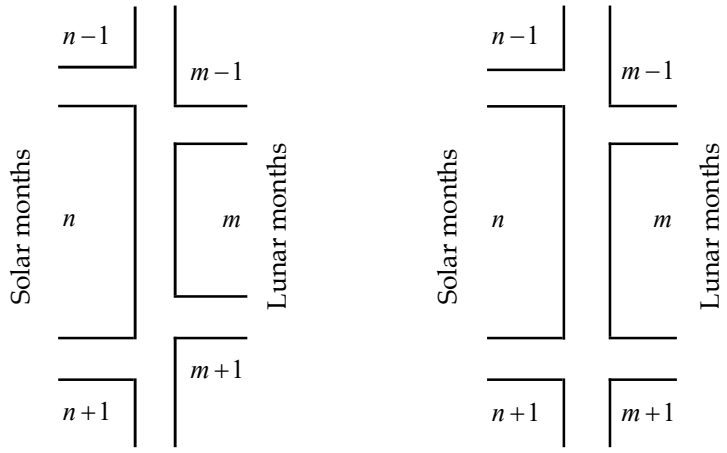
The Chinese lunisolar calendar results from the mixture of the solar calendar, as we have defined it, with the lunar months. The lunar months can only have 29 or 30 days, depending on the time of day when the astronomical phenomenon of the new Moon occurs, while the solar months can have durations of 29, 30, 31, and 32 days.

When combining both types of months, we find three possible situations graphically described in Illustration 3.11. The usual situation is represented in drawing 1 when the solar and lunar months are in the form of a «zipper», that is, in each solar month, there is only the beginning of a lunar month, and in each lunar month, there is only the beginning of a solar month.

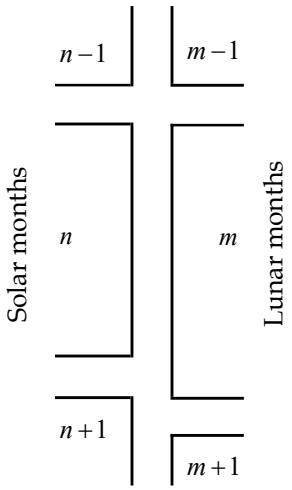


Ilustration 3.11.- **RELATIONSHIP BETWEEN SOLAR AND LUNAR MONTHS**. In the first diagram, we see the usual situation: there is only one major solar term for each lunar month. Then, the numbers of the solar and lunar months coincide. In the second diagram, we have a double lunar month, a lunar month where two major solar terms are included, specifically lunar month n, where the major terms Zn and Zn+1 are. When there is a double lunar month, the coincidence between the numbers of the lunar and solar months is broken. In the third diagram, we have an empty lunar month, which occurs when no major solar term is included in the lunar month; in our case, it is month n. When this circumstance occurs, the numbers of the lunar and solar months do not coincide (unless it is an intercalary month), at least for part of the year.

All intercalary months are empty months, but not all empty months are intercalary. If in a year, whether intercalary or not, there is an empty non-intercalary lunar month (called a fake month), then there must be a double lunar month so that at the end of the year, there are 12 solar months.



Ilustration 4.11.- **EMPTY LUNAR MONTHS**. An empty lunar month does not have any major solar term. The drawing schematically represents the only two possibilities of an empty lunar month. In both cases, the lunar month is entirely included in the solar month.



Ilustration 5.11.- **DOUBLE LUNAR MONTHS**. A double lunar month has two major solar terms. The drawing schematically represents the only possibility of a double lunar month. It occurs when the lunar month has 30 days and the solar month 29.

Since there are lunar months with longer durations than solar months, which occurs when the lunar month has 30 days and the solar month 29, it can happen that in the same lunar month, there are the beginnings of two solar months. We say that the lunar month is double.

Since a lunar month can have a shorter duration than a solar month, the same solar month may contain at the beginning of two lunar months, as shown in Illustration 4.11. We speak of an empty lunar month. Note that intercalary months are always empty lunar months, but not all empty lunar months are intercalary months. We call empty months that are not intercalary fake lunar months.

The double months we have defined before can only occur in the solar months Z1, Z9, Z10, Z11 and Z12 since these are the only ones that can be 29 days long.

In Illustrations 4.11 and 5.11, we schematically show the possibilities of empty and double lunar months. In Illustration 4.11, we show lunar months with no solar term, i.e., empty months. These months are entirely included in the solar month associated with them.

Table 5.11.- **FREQUENCY OF INTERCALARY MONTHS**. Intercalary months may have any of the names, from intercalary-1 to intercalary-12. In the table on the right, we show the frequency of intercalary months between 2025 and 2324, and between 2325 and 2624, we separate this into two groups because of the uncertainty we have about the official time in the distant future. We verify that there is no intercalary month after month 12 in the period considered, although there is no impossibility for this to occur. Thus, the only singular intercalary months (those between months 11 and 1) are the five intercalary-11 that occur in the years 2033, 2128, 2147, 2242 and 2615 (note that the non-intercalary *nian* years are 2034, 2129, 2147, 2243 and 2616). We also see that it is very unlikely that the intercalary-1 month exists; in the period considered it only occurs in the *nian* year 2262. The intercalary-10 month is also very unlikely, occurring in 2166 and 2318. The most probable is the intercalary-5 month, since, as we have seen, it has the longest duration and can most likely include a complete lunar month. We observe that intercalary months are more likely in the summer season.

In Illustration 4.11, we schematically show the only existing possibility of a double lunar month, one with two major solar terms. This situation can only occur with solar months of 29 days and lunar months of 30 days.

The months we have called empty can occur in all solar months because all can have a duration greater than 29 days and, therefore, less than 30 days of a lunar month. However, the probability of a solar month being associated with an empty lunar month differs for each month. For example, solar month Z11 has the lowest probability of not being wholly included in a lunar month because it is the solar month with the shortest duration. Moreover, month Z5 has the highest probability of producing an empty month because it is the longest.

Since intercalary months must be empty lunar months, the frequency of an intercalary month is related to the probability of an empty month. The frequencies of intercalary months are shown in Table 5.11, where we can see a significant difference, finding, as expected, that the summer lunar months 3, 4, 5, 6 and 7 are more frequently intercalary. For the next six hundred years, we have not found any intercalary month 12, which would be an intercalary month immediately before month 1. However, in principle, there is no impediment to this intercalary month.

We would like to point out again that our calculations do not consider future intercalary seconds that may be placed; that is, the time scale we consider is UTC in 2017 when the last intercalary second was placed at the time of writing this book. However, as we have already explained, the calculations will be reliable for at least the next three hundred years.

Intercalary months	From 2024 to 2324		From 2325 o 2624	
	Frequency	%	Frequency	%
1	1	0.90	3	2.73
2	6	5.41	4	3.64
3	14	12.61	13	11.82
4	18	16.22	18	16.36
5	21	18.92	22	20.00
6	21	18.92	17	15.45
7	16	14.41	17	15.45
8	5	4.50	10	9.09
9	3	2.70	1	0.91
10	2	1.80	4	3.64
11	4	3.63	1	0.91
12	0	0	0	0

Years	Month intercalary	Difference	Years	Month intercalary	Difference	Years	Month intercalary	Difference	Years	Month intercalary	Difference
2025	6	2	2099	2	3	2172	5	3	2245	6	3
2028	5	3	2101	7	2	2175	3	3	2248	5	3
2031	3	3	2104	5	3	2177	7	2	2251	3	3
2033	11	2	2107	4	3	2180	6	3	2253	7	2
2036	6	3	2109	9	2	2183	4	3	2256	6	3
2039	5	3	2112	6	3	2186	2	3	2259	5	3
2042	2	3	2115	4	3	2188	6	2	2262	1	3
2044	7	2	2118	3	3	2191	5	3	2264	7	2
2047	5	3	2120	7	2	2194	3	3	2267	5	3
2050	3	3	2123	5	3	2196	7	2	2270	3	3
2052	8	2	2126	4	3	2199	6	3	2272	8	2
2055	6	3	2128	11	2	2202	4	3	2275	6	3
2058	4	3	2131	6	3	2204	9	2	2278	4	3
2061	3	3	2134	5	3	2207	6	3	2281	2	3
2063	7	2	2137	2	3	2210	4	3	2283	6	2
2066	5	3	2139	7	2	2213	3	3	2286	5	3
2069	4	3	2142	5	3	2215	7	2	2289	3	3
2071	8	2	2145	4	3	2218	5	3	2291	7	2
2074	6	3	2147	11	2	2221	4	3	2294	6	3
2077	4	3	2150	6	3	2223	9	2	2297	4	3
2080	3	3	2153	5	3	2226	7	3	2300	2	3
2082	7	2	2156	3	3	2229	5	3	2302	6	2
2085	5	3	2158	7	2	2232	3	3	2305	5	3
2088	8	3	2161	6	3	2234	8	2	2308	3	3
2090	8	2	2164	4	3	2237	5	3	2310	7	2
2093	6	3	2166	10	2	2240	4	3	2313	6	3
2096	4	3	2169	6	3	2242	11	2	2316	4	3

Table 6.11.- ***NIAN* INTERCALARY YEARS.** We show the intercalary months from 2025 to 2316, indicating the position of the intercalary month. We also indicate the years between consecutive intercalary years in the «Difference» column. The difference can only be 3 or 2 years, being more frequent with 3 years. Note that the possible sequences are 2, 3, 3, 3 or 2, 3, 3, with both sequences usually being intercalated.

Solar month	Beginning of the solar month	Ecliptic longitude	Duration solar month	Lunar month	Beginning of the lunar month
2024					
Z1	2024/02/19	330	30	1	2024/02/10
Z2	2024/03/20	360	30	2	2024/03/10
Z3	2024/04/19	30	31	3	2024/04/09
Z4	2024/05/20	60	32	4	2024/05/08
Z5	2024/06/21	90	31	5	2024/06/06
Z6	2024/07/22	120	31	6	2024/07/06
Z7	2024/08/22	150	31	7	2024/08/04
Z8	2024/09/22	180	31	8	2024/09/03
Z9	2024/10/23	210	30	9	2024/10/03
Z10	2024/11/22	240	29	10	2024/11/01
Z11	2024/12/21	270	30	11	2024/12/01
Z12	2025/01/20	300	29	12	2024/12/31
2025					
Z1	2025/02/18	330	30	1	2025/01/29
Z2	2025/03/20	360	31	2	2025/02/28
Z3	2025/04/20	30	31	3	2025/03/29
Z4	2025/05/22	60	31	4	2025/04/28
Z5	2025/06/21	90	31	5	2025/05/27
Z6	2025/07/22	120	32	6	2025/06/25
				intercalary-6	2025/07/25
Z7	2025/08/23	150	31	7	2025/08/23
Z8	2025/09/23	180	30	8	2025/09/23
Z9	2025/10/23	210	30	9	2025/10/21
Z10	2025/11/22	240	29	10	2025/11/20
Z11	2025/12/22	270	30	11	2025/12/20
Z12	2026/01/20	300	29	12	2026/01/19

Table 7.11.- **CHINESE SOLAR CALENDAR**. The previous page represents the solar calendar for 2024 and 2025. The solar month begins at 0 hours on the day the solar term occurs, regardless of the time it occurs. The first month of the Chinese solar calendar is the one with the solar term of ecliptic longitude 330º, which we call Z1. In the table, we indicate the length of the solar month, which can be 29, 30, 31 and 32 days, depending on the distance from the Sun to the perihelion. We also show the lunar month in which the solar month begins. We can see that the numbering of the solar and lunar months coincide, which occurs in most cases, but not always. If an empty lunar month (without a major solar term) is not intercalary, then the correspondence between solar and lunar months is broken. Lunar months begin to be counted from the New Year.
The second table represents the solar calendar 2025, an intercalary year since the *nian* year is 13 months long (and the *sui* year is the same length). The intercalary month follows the lunar month 6 because it does not contain any solar term; that is, it is completely included in a solar month, specifically in the solar month Z6. Since the intercalary month is not numbered, the numbering of solar and lunar months remains the same.

Table 6.11 shows the intercalary years from the year 2025 to the year 2316, where we indicate the month that will be intercalary. In the adjacent column, which we call «Difference», are the years between one intercalary year and the previous one. Between intercalary years, there can be a difference of 3 or 2 years, with 3 years being more likely. Looking at Table 6.11, we find that the succession of intercalary years follows two different patterns: 2, 3, 3, 3 years and 2, 3, 3 years. Usually, these sequences are repeated alternately.

Chinese solar calendar

Table 7.11 shows the solar calendars for the years 2024 and 2025, as examples. Solar months are identified by the major solar term with which they begin, while lunar months are numbered.

Table 7.11 shows the start dates of solar and lunar months. In the year 2024, the lunar and solar months are arranged in a zipper-like fashion, meaning that they follow one another consecutively.

We check that the lunar year 2025 is intercalary since it has 13 solar months; therefore, one of those lunar months must be empty, that is, where no major solar term is found or, in other words, that there is a lunar month that is completely included in a solar month. Indeed, the lunar month that begins on 2026/07/25 and ends on 2026/08/22 occurs within the solar month that begins on 2026/07/22 and ends on 2026/08/22; that is, the solar month has 29 days and the solar month 32.

Table 7.11 shows that the *sui* year 2025 is also intercalary since it is 13 months. In most cases, if the *sui* year is intercalary, the corresponding *nian* year is also intercalary. This rule is broken when there is a singular intercalary month, that is, the intercalary month is between month 11 and month 1, that is, it is intercalary-11 or intercalary-12; something very exceptional as shown by the fact that in the next six hundred years it will only occur in five *nian* years: 2033, 2128, 2147, 2242 and 2615, the *sui* intercalary years being: 2034, 2129, 2147, 2243 and 2616, that is, the years following the *nian* intercalary years.

12.- Singular intercalary months

Frequency of intercalary months

Intercalary months, or the thirteenth month of abundant years, are numbered according to the number of the previous month preceded by the word intercalary. There can be 12 intercalary months, from intercalary-1 to intercalary-12. However, the frequency with which intercalary months occur is very variable.

The velocity of the Sun varies because it follows an elliptical orbit, as described by Kepler's laws of planetary motion. When the Sun's velocity decreases, which happens at aphelion during summer, the solar months become longer. This increase in length results in a higher likelihood that these months will include an entire lunar month. Consequently, intercalary months will be more frequent during the summer.

When the Sun is at perihelion, which occurs during winter, its orbital velocity is greater, and the solar months are shorter and less likely to be intercalary.

In Table 1.12, we analyze the frequency of intercalary months for the period from 2025 to 2624. As anticipated, intercalary months are more likely to occur during lunar months 3 to 7 (approximately April to August) compared to the remaining months, which take place in autumn and winter. We find a significant difference in frequency: there is an 86% probability that an intercalary month will fall within the group of six months from 3 to 8, while the probability of it occurring in the remaining six months is only 14%.

In Table 1.12 there is no intercalary month 12 in the analyzed period. However, such a month may exist. In the year 3000, the average duration of solar month 12 is 29.50 days, meaning that month Z12 could last 29 or 30 days. Similarly, lunar months can also have durations of 29 or 30 days. Therefore, solar month Z12 can extend for 30 days while the corresponding lunar month lasts only 29 days. In this scenario, the lunar month might not include any solar terms, qualifying it as an intercalary month.

Helmer Aslaksen claims there will be an intercalary-12 month in 3358. We find an intercalary month after month 1 of 3359, i.e. intercalary-1, provided we use the UTC(2017) scale, i.e. without adding intercalary seconds in the future, keeping only the intercalary seconds placed up to 2017. However, if we consider the expected intercalary seconds us to 3359, which we can calculate by (3.4) and add up to 2 hours and 7 minutes, then the intercalary month will be the 12th of the year 3358, as explained in Table 2.12.

In this example, we find that we cannot extrapolate the Chinese calendar into the distant future because of the uncertainty about the civil time scale.

Intercalary months	From 2024 to 2324		From 2325 to 2624	
	Frequency	%	Frequency	%
1	1	0.90	3	2.73
2	6	5.41	4	3.64
3	14	12.61	13	11.82
4	18	16.22	18	16.36
5	21	18.92	22	20.00
6	21	18.92	17	15.45
7	16	14.41	17	15.45
8	5	4.50	10	9.09
9	3	2.70	1	0.91
10	2	1.80	4	3.64
11	4	3.63	1	0.91
12	0	0	0	0

Table 1.12.- **FREQUENCY OF INTERCALARY MONTHS**. Intercalary months may have any of the names from intercalary-1 to intercalary-12. The table shows the frequency of intercalary months between 2024 and 2324 and between 2325 and 2624. We verify that there is no intercalary month after month 12 in the period considered. Thus, the only singular intercalary months are the four intercalary-11 that occur in 2033, 2128, 2147 and 2242 (Table 6.11). It is improbable that an intercalary month-1 exists, which occurs in the first period considered in the year 2262. The intercalary month-10 is also very unlikely, occurring in 2166 and 2318. The most likely are the intercalary months 5 and 6. We observe that intercalary months are more likely in the summer because the Sun is in the aphelion zone where its velocity is lower and the solar months are longer.
We point out again that the calculations refer to civil time UTC(2017); that is, we only consider intercalary seconds placed up to the year 2017. If the intercalary second system continued, there would be approximately 2 hours accumulated in 3358.

Singular intercalary months

We call the 11th and 12th intercalary months singular intercalary months because their infrequent occurrence significantly alters the calendar. From Table 6.11, we find that in the next three hundred years, 2033, 2128, 2147, and 2242 will have intercalary-11 months.

When there is a singular intercalary month, the corresponding *sui* year will not be intercalary, but the *nian* year will be. The following year, the *sui* will be intercalary, and the *nian* year will not be intercalary.

In the past, some astronomers believed there could not be any singular intercalary months. Until the 1990s, all Chinese calendars made an error with the year 2033 since they assumed that the intercalary month was the 8th, not the 11th (Table 3.12). However, the eighth month is not intercalary since it belongs to the common *sui* year 2033, which has only 12 months. In the *nian* year 2033, the intercalary

UTC(2017)		UTC(3359)		Solar term UTC(2017) and UTC(3359)
Month	New Moon	Month	New Moon	
10	3358/10/25 - 06:10	10	3358/10/25 - 04:03	3358/11/22
11	3358/11/23 - 17:01	11	3358/11/23 - 14:54	3358/12/22
12	3358/12/23 - 03:41	12	3358/12/23 - 01:34	3359/01/20
1	3359/01/21 - 14:31	12i	3359/01/21 - 12:24	3359/02/19
1i	3359/02/20 - 01:26	1	3359/02/19 - 23:19	3359/03/21
2	3359/03/21 - 14:24	2	3359/03/21 - 10:17	3359/04/20
3	3359/04/19 - 23:38	3	3359/04/19 - 21:31	3359/05/20

Table 2.12.- **POSSIBLE INTERCALARY MONTH-12 IN 3358**. We see in Table 1.12 that there will not be an intercalary-12 month in the next six hundred years as long as we consider that there will be no new intercalary seconds. In the table's UTC(2017) column are the dates of the new Moons at the end of 3358 and the beginning of 3359, calculated if civil time does not include new intercalary seconds. Comparing the solar terms in the last column, we see that the month that begins on 3359/02/20 is intercalary, as it does not have any solar terms. So the year 3359 will be intercalary, and the intercalary month will be 1. In the UTC(3359) column are the dates of the new Moons but calculated with the civil time that would result from placing intercalary seconds as has been done until now. We see that, in this case, the intercalary month will be the one that begins on 3359/01/19. So the year 3358 will be abundant, and the intercalary month will be the 12th.
This example illustrates that calculating the Chinese calendar for the distant future can result in errors due to our uncertainty about the future civil time scale.

month is the 11th, which belongs to the intercalary *sui* year 2034, which is a year that begins immediately after the 11th and has 13 months.

Singular years

Table 3.12 shows the calendar for the years 2033/2034, 2128/2129, 2147/2148, and 2242/2243. The four calendars considered in Table 3.12 also have the characteristics that each has an empty non-intercalary month or fake month and a double month or one with two major solar terms.

The calculations use UTC(2017) as the civil time scale, meaning no new intercalary seconds will be inserted except those already inserted up to 2017. Given the imminent change in the civil time scale, we are in favour of adopting the most reasonable measure: from a certain date, no more intercalary seconds will be inserted, but those that have already been inserted will continue.

In Table 3.12, we show singular years with the intercalary-11 months. However, there are other years with other singular months. In Table 4.12, we show the year 1984/1985, where we find a fake month and a double month, but the intercalary month is 10. In Table 5.12, we put part of the years 1851/1852, where we show the pairing of a fake month and a double month, with the intercalary month being 8.

To avoid errors with the singular months, we determine an intercalary year by the *sui* year, the one that begins immediately after the 11th month and is characterized by having the winter solstice. If this *sui* year has 12 months, it is not intercalary, and therefore, if there is an empty month, as in the examples shown, it must be a fake month, that is, empty but not intercalary.

From the examples provided, it is evident that fake months are linked to double months, which are necessary for the calendar to have 12 solar months. In double months, there are always two major solar terms and at least one minor solar term. Additionally, it is possible for a lunar month to have two minor solar terms and only one major solar term; however, we will not consider this situation, as it does not affect the creation of the calendar.

Before the Jesuit reform of 1645, which established the modern calendar, double months or months with two major solar terms were impossible. As we see in Illustration 4.11, for a lunar month to be double, it must have a duration of 30 days, and the solar month must be 29 days. When using mean values of the Sun, as occurred before the Jesuit reform, the solar month always had a duration of 30.44 day $(= 30 \times 365.25 / 360)$, that is, the solar month could have 30 or 31 days, but never 29 days.

The introduction in 1645 of the true motion of the Sun instead of the mean motion for calendar calculations gave rise to problems such as those described in this section, which would not have occurred if solar months could not have 29 days, as occurs in the current calendar.

As we have seen, double lunar months can only occur in association with solar months of 29 days; for this to happen, the solar month must be less than 30

Tables 3.12.- **CALENDARS FOR THE YEARS 2033/2034, 2128/2129, 2147/2148 AND 2242/2243.** The following four tables show the calendars for the next years whose intercalary month is the 11th, that is, 2033, 2128, 2147 and 2242.

The year *sui* 2033 is a common year of 12 lunar months because from month 11 (the one containing the winter solstice) of 2032 to month 11 of 2033, there are twelve months. The year sui 2034 will be embolismic because there are 13 months from month 11 of 2033 to month 11 of 2034. The eighth month of 2033 (which begins on August 25, 2033) is a fake month because it has no major solar term and is not an intercalary month since the year *sui* 2033 is common

Month 11 of 2033 (starting on 2033/11/22) and month 1 of 2034 (starting on 2034/01/20) have the particularity that they contain two major solar terms. The intercalary month of 2033 is the one that comes after month 12 (starting on 2033/12/22) since it is the first month of that year that does not have major solar terms. Month 2034/02/19 is another fake month because although it does not have major solar terms, it is not intercalary since the intercalary month is the one that begins on 2033/12/22, which is the first empty month of the *sui* year 2033. Despite the oddities of these years, it fully complies with the rules of the Chinese calendar.

The calendars for the years 2128/2129, 22147/2148 and 2242/2243 have similar characteristics to the previous one: the intercalary month is the 11th, and all of them have an empty month (not intercalary or fake) and a double month (with two solar terms in the lunar month).

In the calendars for these years, lunar months are numbered from 1 to 12, similar to solar months. However, it is important to note that the lunar month n does not always correspond with the solar month Zn. In particular, in 2034 and 2146, the lunar month in which the year nian begins (or month 1) does not correspond to the solar month Z1.

New Moon. Beginning of the lunar month	Number lunar month	Days of the lunar month	Date of major solar terms	
2032/12/03	11	29	2032/12/21	Z11-2032
2033/01/01	12	30	2033/01/20	Z12
2033/01/31	1	29	2033/02/18	Z1
2033/03/01	2	30	2033/03/20	Z2
2033/03/31	3	29	2033/04/20	Z3
2033/04/29	4	29	2033/05/21	Z4
2033/05/28	5	30	2033/06/21	Z5
2033/06/27	6	29	2033/07/22	Z6
2033/07/26	7	30	2033/08/23	Z7
2033/08/25	8	29	....	....
2033/09/23	9	30	2033/09/23	Z8
2033/10/23	10	30	2033/10/23	Z9
2033/11/22	11	30	2033/11/22	Z10
....	....	....	2033/12/21	Z11-2033
2033/12/22	11i	29	....	....
2034/01/20	12	30	2034/01/20	Z12
....	....	....	2034/02/18	Z1
2034/02/19	1	29	....	....
2034/03/20	2	30	2034/03/20	Z2
2034/04/19	3	29	2034/04/20	Z3
2034/05/18	4	29	2034/05/21	Z4
2034/06/16	5	30	2034/06/21	Z5
2034/07/16	6	29	2034/07/23	Z6
2034/08/14	7	30	2034/08/23	Z7
2034/09/13	8	29	2034/09/23	Z8
2034/10/12	9	30	2034/10/23	Z9
2034/11/11	10	30	2034/11/21	Z10
2034/12/11	11	29	2034/12/22	Z11-2034

New Moon. Start of the lunar month	Number lunar month	Days of the lunar month	Date of major solar terms	
2127/12/04	11	29	2127/12/22	Z11-2127
2128/01/02	12	30	2128/01/21	Z12
2128/02/01	1	29	2128/02/19	Z1
2128/03/01	2	30	2128/03/20	Z2
2128/03/31	3	29	2128/04/20	Z3
2128/04/29	4	30	2128/05/21	Z4
2128/05/29	5	30	2128/06/21	Z5
2128/06/28	6	29	2128/07/22	Z6
2128/07/27	7	30	2128/08/23	Z7
2128/08/26	8	29	2128/09/23	Z8
2128/09/24	9	30	2128/10/23	Z9
2128/10/24	10	29	....	....
2128/11/22	11	30	2128/11/22	Z10
....	....	....	2128/12/21	Z11-2128
2128/12/22	11i	29	.....	....
2129/01/20	12	30	2129/01/20	Z12
2129/02/19	1	29	2129/02/19	Z1
2129/03/20	2	30	2129/03/20	Z2
2129/04/19	3	29	2129/04/19	Z3
2129/05/18	4	30	2129/05/21	Z4
2129/06/17	5	29	2129/06/21	Z5
2129/07/16	6	30	2129/07/23	Z6
2129/08/15	7	30	2129/08/23	Z7
2129/09/14	8	29	2129/09/23	Z8
2129/10/13	9	30	2129/10/23	Z9
2129/11/12	10	29	2129/11/22	Z10
2129/12/11	11	30	2129/12/22	Z11-2129
2130/01/10	12	29	2130/01/20	Z12

New Moon. Start of the lunar month	Number lunar month	Days of the lunar month	Date of major solar terms	
2146/12/04	11	29	2146/12/22	Z11-2146
2147/01/02	12	30	2147/01/20	Z12
2147/02/01	1	29	2147/02/19	Z1
2147/03/02	2	29	2147/03/21	Z2
2147/03/31	3	30	2147/04/20	Z3
2147/04/30	4	29	2147/05/21	Z4
2147/05/29	5	30	2147/06/21	Z5
2147/06/28	6	30	2147/07/23	Z6
2147/07/28	7	29	2147/08/23	Z7
2147/08/26	8	30	2147/09/23	Z8
2147/09/25	9	30	2147/10/24	Z9
2147/10/25	10	29	2147/11/22	Z10
2147/11/23	11	30	2147/12/22	Z11-2147
2147/12/23	11i	29	....	....
2148/01/21	12	30	2148/01/21	Z12
....	....	....	2148/02/19	Z1
2148/02/20	1	29	....	....
2148/03/20	2	30	2148/03/20	Z2
2148/04/19	3	29	2148/04/19	Z3
2148/05/18	4	29	2148/05/20	Z4
2148/06/16	5	30	2148/06/21	Z5
2148/07/16	6	29	2148/07/22	Z6
2148/08/14	7	30	2148/08/22	Z7
2148/09/13	8	30	2148/09/22	Z8
2148/10/13	9	29	2148/10/23	Z9
2148/11/11	10	30	2148/11/22	Z10
2148/12/11	11	30	2148/12/21	Z11-2148
2149/01/10	12	29	2149/01/20	Z12

New Moon. Start of the lunar month	Number lunar month	Days of the lunar month	Date of major solar terms	
2241/12/04	11	29	2241/12/22	Z11-2242
2242/01/02	12	30	2242/01/20	Z12
2242/02/01	1	29	2242/02/19	Z1
2242/03/02	2	30	2242/03/21	Z2
2242/04/01	3	30	2242/04/20	Z3
2242/05/01	4	29	2242/05/21	Z4
2242/05/30	5	30	2242/06/21	Z5
2242/06/29	6	30	2242/07/23	Z6
2242/07/29	7	29	2242/08/23	Z7
2242/08/27	8	29	2242/09/23	Z8
2242/09/25	9	30	2242/10/24	Z9
2242/10/25	10	29	....	....
2242/11/23	11	30	2242/11/23	Z10
....	....	....	2242/12/22	Z11-2242
2242/12/23	11i	29	....	....
2243/01/21	12	30	2243/01/21	Z12
2243/02/20	1	29	2243/02/19	Z1
2243/03/21	2	30	2243/03/21	Z2
2243/04/20	3	29	2243/04/20	Z3
2243/05/19	4	30	2243/05/21	Z4
2243/06/18	5	30	2243/06/22	Z5
2243/07/18	6	29	2243/07/23	Z6
2243/08/16	7	30	2243/08/23	Z7
2243/09/15	8	29	2243/09/23	Z8
2243/10/14	9	30	2243/10/14	Z9
2243/11/13	10	29	2243/11/13	Z10
2243/12/12	11	30	2243/12/12	Z11-2243
2244/01/11	12	29	2244/01/11	Z12

New Moon. Start of the lunar month	Number lunar month	Days of the lunar month	Date of major solar terms	
1983/12/04	11	30	1983/12/22	Z11-1983
1984/01/03	12	30	1984/01/21	Z12
1984/02/02	1	30	1984/02/19	Z1
1984/03/03	2	29	1984/03/20	Z2
1984/04/01	3	30	1984/04/20	Z3
1984/05/01	4	30	1984/05/21	Z4
1984/05/31	5	29	1094/06/21	Z5
1984/06/29	6	29	1984/07/22	Z6
1984/07/28	7	30	1984/08/23	Z7
1984/08/27	8	29	1984/09/23	Z8
1984/09/25	9	29	1984/10/23	Z9
1984/10/24	10	30	1984/11/22	Z10
1984/11/23	10i	29	....	....
1984/12/22	11	30	1984/12/22	Z11-1984
1985/01/21	12	30	1985/01/21	Z12
....	....	....	1985/02/19	Z1
1985/02/20	1	29	....	....
1985/03/21	2	30	1985/03/21	Z2
1985/04/20	3	30	1985/04/20	Z3
1985/05/20	4	29	1985/05/21	Z4
1985/06/18	5	30	1985/06/21	Z5
1985/07/18	6	29	1985/07/23	Z6
1985/08/16	7	30	1985/08/23	Z7
1985/09/15	8	29	1985/09/23	Z8
1985/10/14	9	29	1985/10/23	Z9
1985/11/12	10	30	1985/11/22	Z10
1985/12/12	11	29	1985/12/22	Z11-1985
1986/01/10	12	30	1986/01/20	Z12

Table 4.12.- **CALENDAR FOR 1984/1985**. On the previous page are the years 1984/1985. 1984 was an intercalary year; the *sui* and *nian* years were 13 months long. The intercalary month was the 10i because the lunation that began on 1984/11/23 had no solar term. The month that began on 1985/01/21, the last month of the *nian* year 1984, was a double month because it had two solar terms. The first lunar month of 1985, which began on 1985/02/20, was a month without a solar term, but it was not an intercalary month because both the *sui* and *nian* years 1985 had 12 months. Therefore, that month was fake.
There are 12 solar terms yearly, but there may be 12 or 13 lunar months. The month in which the winter solstice occurs is always Z11. Except in exceptional cases, the first month of the year *nian*, whose month has the designation 1, corresponds to the solar term Z1.

New Moon. Start of the lunar month	Number lunar month	Days of the lunar month	Date of major solar terms	
1851/11/23	11	29	1851/11/23	Z11-1851
1851/12/22	12	30	1851/12/22	Z12
1852/01/21	1	30	1852/01/21	Z1
....	....	....	1852/02/19	Z2
1852/02/20	2	30	1852/03/20	Z3
1852/03/21	3	29	....	....
1852/04/19	4	30	1852/04/20	Z4
1852/05/19	5	30	1852/05/21	Z5

Table 5.12.- **CALENDAR FOR 1851/1852**. In the *nian* year 1851, the intercalary month was 8. In the following *nian* year, there was a succession of a double month (with two major solar terms) and an empty month (without solar terms), which, as that year was not intercalary, is a fake and non-intercalary month.

days. From Table 1.11, we see that for the current period, only solar months Z1, Z9, Z10, Z11, and Z12 can belong to double lunar months, as we can confirm in Table 3.12.

If there is a double month, there must be an empty or fake month to compensate and vice versa; this is mandatory because there must be 12 solar months in 365 or 366 days. The fake month may belong to the same lunar year (this happens in the years 2128 and 1985) or to different lunar years (years 2034, 2148 and 2242), but always within a period of 366 or 365 days. Since double lunar months could not exist in the old calendar system, there could not be fake months either.

In the old calendar that used the mean motion of the Sun, there could be empty months, which occurred when the solar month had 31 days and the lunar month 29 or 30, or the solar month had 30 days and the lunar month had 29, but these months had to be intercalary. Therefore, in the calendar before 1645, all empty months were intercalary.

Another noteworthy circumstance is the occurrence of double spring years, including the minor solar term J1 at the beginning and end of the year. This can only occur in intercalary years since a double spring year must have at least 366 days.

Note that the minor solar term J1 occurs on the 3rd, 4th, or 5th of February. New Year's day falls between January 21st and February 21st, both inclusive, which means the J1 term can occur at the year's beginning and end. The number of days between two successive J1 terms, including both dates, can range from 364 to 369. Therefore, a double spring year must be intercalary to account for these days; however, not all intercalary years are double spring years *.

A year is double spring-double rain if it contains the solar terms J1 and Z1, both at the beginning and at the end of the year, a characteristic that is significant in Chinese astrology. Naturally, a double spring-double rain year must be intercalary.

A blind year is defined as one that does not start with the term J1 but does end with it. Additionally, a blind year must be a common year. This is because, even if the lunisolar year begins on the latest possible date, February 21, the time elapsed from J1 to the end of the lunisolar year would not exceed the length of an intercalary year.

We call it a double-blind year when the lunisolar year does not have the J1 term either at the beginning or the end. If the year has the J1 at the beginning but not at the end, it is called a bright year.

There can only be a singular intercalary month (i.e. one that falls between months 11 and 1) if there is a new Moon very soon after, but on different days, the winter solstice. Then, from that new Moon to the beginning of the following year, two months could fit, one of which would be the intercalary month. Note that a singular intercalary month does not mean the beginning of the New Year is delayed.

* Suppose the *nian* year begins on February 3, the same day as J1. Then, the New Year of the following year will be approximately 11 days earlier (Chapter 14), i.e., on January 23, and therefore, it cannot contain J1.

13.- Meton cycle and the Chinese calendar

Meton cycle

Chapter 6 discusses the Meton cycle used in computational calendars. It is based on the equivalence of 19 solar years and 235 lunar cycles. This coincidence is accidental, as lunar and solar movements are independent phenomena.

The Meton cycle is an astronomical relationship, so, logically, we find it in the Chinese astronomical calendar. In Table 1.13, we analyze the 19 years from 2024 to 2042, and we find some of the characteristics set out in Chapter 6, among them that in 19 years, there are 7 intercalary years; the cycle has a duration of 6,940 days, the order of intercalation is 2, 5, 8, 10, 13, 16 and 19, as we had found in Chapter 6, in addition, the total number of months of 30 days is 125, and the number of months of 29 days is 110, a total of 235 lunations. The choice of the first year of the cycle is arbitrary; in Table 1.13, we have chosen the year 2024.

The Meton cycle, when applied to computational calendars, assumes that all intercalary months have 30 days. However, this assumption does not hold true for the Chinese calendar, as shown in Table 1.13. In the Chinese calendar, intercalary months can be either 29 or 30 days long. As a result, the number of non-intercalary months with 29 and 30 days differs from the Meton cycle as applied to the Julian calendar. Specifically, there are 119 non-intercalary months with 30 days in the Chinese calendar, while the Meton cycle for computational calendars indicates there should be only 118.

In Table 2.13, we analyze 36 Meton cycles of the Chinese calendar, starting in 2024. We see that during this interval, 7 of each 19-year cycle are intercalary. The Meton cycle applies to both *sui* and *nian* years, although there are some differences between them. In addition, the order of intercalation is the same as that deduced in Chapter 6.

As we explained in Chapter 6, for the Meton cycle to fit the Julian year of 365.25 days, some cycles must be 6,940 days long and others 6,939; this is shown in Table 2.13, where we also find a cycle with 6,941 days.

In Table 3.13, we put the dates on which 26 Meton cycles begin in the *sui* year, finding that these dates are approximately coincidental. However, the Meton cycle is not strictly followed, as we observe in Table 3.13, where the beginning of the cycle arrives later and later. In Table 4.13, we put 26 Meton cycles for the *nian* year, again finding the same results as with the *sui* years.

There is an approximate coincidence between Gregorian and Chinese dates

Num.	Year	Common/ Intercalar	Duration year	Non-intercalary months		Intercalary month duration	Intercalary month position
				30 days	29 days		
1	2024	C	354	6	6	-	-
2	2025	I	384	6	6	30	6i
3	2026	C	354	6	6	-	-
4	2027	C	354	6	6	-	-
5	2028	I	384	6	6	30	5i
6	2029	C	355	7	5	-	-
7	2030	C	354	6	6	-	-
8	2031	I	384	6	6	30	3i
9	3032	C	355	7	5	-	-
10	3033	I	384	6	6	30	11i
11	2034	C	354	6	6	-	-
12	2035	C	354	6	6	-	-
13	2036	I	384	7	5	29	6i
14	2037	C	354	6	6	-	-
15	2038	C	354	6	6	-	-
16	2039	I	384	6	6	30	5i
17	2040	C	355	7	5	-	-
18	2041	C	355	7	5	-	-
19	2042	I	384	6	6	30	2i
			6,940	119	109		

Table 1.13.- **METON CYCLE 2024/2042.** We analyze the 19 years from 2024 to 2042. We check that it fits the Meton cycle. In the first column are the order numbers; we chose 2024 as the first year of the cycle. In the 19 years, there are 7 that are intercalary; the total duration of the cycle is 6,940 days; there are 109 standard months (that is, not intercalary) of 30 days and 119 standard months of 29 days. Finally, we see that the sequence of the intercalary years is 2, 5, 8, 10, 13, 16, and 19, an order that, in Chapter 6, we have called $\beta = 17$ (Table 2.6). Some intercalary months have 29 days, and others 30. We see some differences with respect to the Meton cycle of the computational calendar that follows the Julian year (Chapter 6), where the intercalary months are all 30 days long, there are 118 standard months of 30 days and 110 typical months of 29 days. The average length of the solar year is 365.26 days (6,940/19), and the average lunation is 29.53 days.

Cycle of 19 years	Position of intercalary years (*sui*)	Days	Position of intercalary years (*nian*)	Days
2024 - 2042	2, 5, 8, 11, 13, 16, 19	6 940	2, 5, 8, 11, 13, 16, 19	6 940
2043 - 2061	2, 5, 8, 10, 13, 16, 19	6 940	2, 5, 8, 10, 13, 16, 19	6 939
2062 - 2080	2, 5, 8, 10, 13, 16, 19	6 939	2, 5, 8, 10, 13, 16, 19	6 940
2081 - 2099	2, 5, 8, 10, 13, 16, 19	6 939	2, 5, 8, 10, 13, 16, 19	6 939
2100 - 2118	2, 5, 8, 11, 13, 16, 19	6 940	2, 5, 8, 10, 13, 16, 19	6 941
2119 - 2137	2, 5, 8, 11, 13, 16, 19	6 940	2, 5, 8, 11, 13, 16, 19	6 940
2138 - 2156	2, 5, 8, 10, 13, 16, 19	6 940	2, 5, 8, 11, 13, 16, 19	6 940
2157 - 2175	2, 5, 8, 11, 13, 16, 19	6 940	2, 5, 8, 10, 13, 16, 19	6 940
2176 - 2194	2, 5, 8, 10, 13, 16, 19	6 939	2, 5, 8, 11, 13, 16, 19	6 939
2195 - 2213	2, 5, 8, 10, 13, 16, 19	6 940	2, 5, 8, 10, 13, 16, 19	6 940
2214 - 2232	2, 5, 8, 11, 13, 16, 19	6 939	2, 5, 8, 11, 13, 16, 19	6 940
2233 - 2251	2, 5, 8, 11, 13, 16, 19	6 940	2, 5, 8, 11, 13, 16, 19	6 940
2252 - 2270	2, 5, 8, 11, 13, 16, 19	6 940	2, 5, 8, 11, 13, 16, 19	6 940
2271 - 2289	2, 5, 8, 11, 13, 16, 19	6 940	2, 5, 8, 11, 13, 16, 19	6 940
2290 - 2308	2, 5, 8, 10, 13, 16, 19	6 940	2, 5, 8, 11, 13, 16, 19	6 940
2309 - 2327	2, 5, 8, 11, 13, 16, 19	6 939	2, 5, 8, 10, 13, 16, 19	6 940
2328 - 2346	2, 5, 8, 11, 13, 16, 19	6 939	2, 5, 8, 11, 13, 16, 19	6 939
2347 - 2365	2, 5, 8, 11, 13, 16, 19	6 940	2, 5, 8, 11, 13, 16, 19	6 940

Table 2.13.- **THE METON CYCLE IN THE CHINESE CALENDAR.** The Chinese calendar follows the Meton cycle very closely, according to which every 19 years, 7 of them are intercalary. The table shows 19-year cycles, which we begin to count from 2024. We see that the succession of intercalary years in each cycle are very similar, only differing in that in some years, the intercalary is the tenth and in other cycles the eleventh, finding an average value of 6,939.67 days, which corresponds to 235 lunations with an average duration of 29.5305 days, which coincides with a difference of only 7.7 seconds with the average value of the synodic period of the Moon. Note that there is some minor difference between the *sui* and *nian* years. If the Meton cycle is in bold, it means there are differences in the cycle of intercalary years in the *sui* and *nian* years. If the years of the cycle are underlined, there are differences in the duration of the cycle.

If another year is chosen to begin counting the Meton cycles, we find another succession of intercalary cycles, but they only differ in order; that is, they have the same intercalation rules but are displaced from each other.

Meton cycle	Gregorian date start of year *sui*	Meton cycle	Gregorian date start of year *sui*
1	2024/01/11	14	2271/01/12
2	2043/01/11	15	2290/01/12
3	2062/01/11	16	2309/01/12
4	2081/01/10	17	2328/01/13
5	2100/01/10	18	2347/01/11
6	2119/01/11	19	2366/01/12
7	2138/01/11	20	2385/01/12
8	2157/01/11	21	2404/01/13
9	2176/01/12	22	2423/01/13
10	2195/01/11	23	2442/01/12
11	2214/01/12	24	2461/01/12
12	2333/01/11	25	2480/01/12
13	2252/01/12	26	2499/01/12

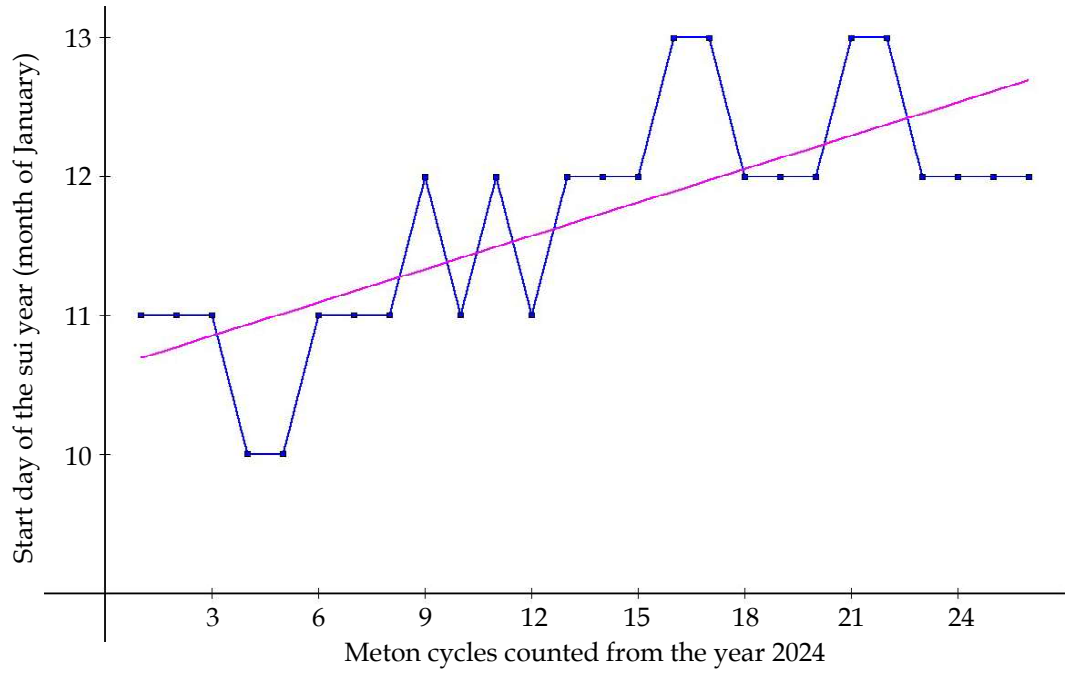
Table 3.13.- **THE METON CYCLE IN THE CHINESE CALENDAR (*SUI*)**. The table shows the Gregorian calendar dates for the day the Meton cycles begin, starting in 2024. If the Meton cycle were fulfilled exactly, the dates should be the same; however, there is a slight variation, in the sense that the beginning of the first year of the Meton cycle is gradually later, proof that 235 lunations do not coincide exactly with 19 Gregorian years. However, the winter equinox, which always occurs the month before the beginning of the *sui* year, has a very slight variation concerning the Gregorian calendar.
The variations observed from one cycle to another are caused by the irregular intercalation of leap days in the Gregorian calendar.

every 19 years. Therefore, the annual anniversaries occur on approximately the same dates in the Chinese calendar every 19 years.

Accumulated error with respect to the solar year

The Meton cycle is not exact because neither the lunation nor the year deduced from the cycle coincides with the astronomical values; therefore, the dates of the true new Moons are displaced by the solar year. We then calculate the average accumulated error after N Meton cycles, i.e. the displacement of the date of the astronomical new Moons in the solar calendar.

If 19 solar years had the same length as 235 astronomical lunations, then the dates of the new Moons located at the same position in the cycle would coincide on the solar calendar. If there were no coincidence between 19 years and 235 lunations,



Ilustration 1.13.- **THE METON CYCLE IN THE CHINESE CALENDAR (*SUI*)**. In the graph, we represent Table 2.11. On the horizontal axis are the Meton cycles, starting with the cycle beginning in 2024. On the vertical axis is the beginning of the *sui* year of the first year of the Meton cycle. We see that the first day of the *sui* year of the first year of the cycle comes later as time goes by. The straight line is the linear fit, with a slope of 0.08 days for every 19 years, coinciding with the result deduced in the text.
The Gregorian year is 365.2425 days, and the mean lunation is 29.53058885 days, so in a Meton cycle, the difference between 19 years and 235 lunations is 0.0806 days—the same as shown in the graph. 235 lunations exceeds 19 Gregorian years, so the date of the new Moons is delayed.

then there would be a shift in the dates of the new Moons at the same position in the cycle.

However, the average solar year y_m is not usually an integer, so the solar calendar will have intercalary years; this means that the calendar year will have different lengths, which will affect the position of the dates of the astronomical new Moons in the solar calendar. However, we will deduce the average error or displacement, assuming that all the solar calendar years have the same length, even if this is a decimal number.

We note that the dates of the new Moons that we deduce from the lunisolar cycle are the average dates since we only use the mean lunation and the mean length of the year, not the true values.

l_m is the mean astronomical lunation and y_m is the mean length of the solar calendar year, then assuming that the length of the lunation does not vary during a cycle, the accumulated error in a single cycle is $(A=19\ ;\ L=235)$

$$E_c = L\,l_m - A y_m\,, \tag{1.13}$$

if y_m could be the Julian or Gregorian year, we would measure how the date of the

Meton cycle	Gregorian date of the New Year	Meton cycle	Gregorian date of the New Year
1	2024/02/10	14	2271/02/11
2	2043/02/10	15	2290/02/11
3	2062/02/09	16	2309/02/11
4	2081/02/09	17	2328/02/12
5	2100//02/09	18	2347/02/11
6	2119/02/09	19	2366/02/11
7	2138/02/10	20	2385/02/10
8	2157/02/10	21	2404/02/11
9	2176/02/11	22	2423/02/11
10	2195/02/10	23	2442/02/11
11	2214/02/10	24	2461/02/11
12	2233/02/10	25	2480/02/12
13	2252/02/11	26	2499/02/10

Tabla 4.13.- **THE METON CYCLE IN THE CHINESE CALENDAR (*NIAN*)**. The table shows the Gregorian dates of the Chinese calendar New Year calculated in 19-year intervals, starting with 2024. We can see that the dates of the New Year are approximately the same (February 9, 10, 11, or 12) as reflected in the Meton cycle. As we have already seen in Table 2.13, there is a gradual delay in the beginning of the New Year as a result of the Meton cycle not being perfectly adjusted to astronomy.

astronomical mean new Moon shifts according to the Julian or Gregorian calendar.

Expressing the lunation as a function of the year (and taking only the first power of time) by (9.5)

$$l_m = a + bT = a + b\frac{Y-2000}{100} = (a-20b) + 0.01bY = a' + b'Y \qquad (2.13)$$

T is the Julian centuries (= 36,525 days) that have elapsed since the beginning of the year 2000, and Y is the year of the common era for which we calculate the durations of the lunation. (2.13) is expressed in units of Universal Time, and in these units y_m is constant. By formulas (1.13) and (2.13)

$$E_c = (La' - Ay_m) + Lb'Y_0,$$

Y_0 is the year the cycle begins, which is arbitrary.

The accumulated error in N cycles of A years is

$$E_c(N) = (La' - Ay_m)N + Lb'\{Y_0 + [Y_0 + A] + [Y_0 + 2A] + \ldots + [Y_0 + (N-1)A]\} =$$

$$= \left(La' - Ay_m\right)N + Lb'Y_0N + \frac{A}{2}Lb'N\left(N-1\right) = \left(La' + Lb'Y_0 - Ay_m\right)N + \frac{A}{2}Lb'N\left(N-1\right) =$$

$$= \left(Ll_{m0} - Ay_m\right)N + \frac{A}{2}Lb'N\left(N-1\right) \tag{3.13}$$

Y_0 is the year from which we calculate the shift of lunations, and l_{m0} is the astronomical mean lunation in the year Y_0. We assume that in the cycle the lunation remains unchanged. The mean lunation en UT es by (7.4)

$$l_m = 29^{\mathrm{d}}.530\ 595\ 63 - 3^{\mathrm{d}}.920\ 892 \cdot 10^{-9} Y.$$

Applying equation (3.13) to the Meton cycle beginning in 2024, we find that the average accumulated error per cycle, in relation to the Gregorian year, is

$$E_c = 0^{\mathrm{d}}.080\ 61 = 1^{\mathrm{h}}\ 56^{\mathrm{m}}. \tag{4.13}$$

Since the lunation is practically constant, E_c remains unchanged. The result (4.13) means that the new Moon's dates are increasingly later in the solar calendar. From (4.13), we also deduce that every 12.4 Meton cycle, the accumulated error will be one day.

In Illustration 1.13, we graphically represent the results of Table 3.13 and see the shift in the dates on which the Meton cycles begin. Using the straight line of fit, we find that every 19 years, there is a shift of 0.08 days, coinciding with the average value found theoretically in (4.13).

14.- Miscellany

Length of lunisolar years

In the Chinese calendar, there are years with 12 months and years with 13 months. The abundant year will have approximately 30 days more than the common year. However, the distribution of months with 29 or 30 days can vary between years of the same type, resulting in years of different lengths.

In addition, we have considered two types of years, the *nian* and the *sui*, which differ in the date on which they begin, although both are lunisolar years, composed of 12 or 13 lunar months, and can have different durations.

In Table 1.14, we have calculated the lengths of *sui* and *nian* years from 2024 to 2324. The possible lengths of these years are 354, 355, 383, 384, and 385 days. While

Days of the year	*Sui*		*Nian*	
	Frequency	%	Frequency	%
353	0	0	0	0
354	122	40.53	124	41.20
355	68	22.59	66	21.93
383	12	3.99	10	3.32
384	97	32.23	99	32.89
385	2	0.66	2	0.66

Table 1.14.- **LENGTH OF YEARS**. The table displays the lengths of the *sui* and *nian* years from 2024 to 2324. During this period, no year is recorded with a length of 353 days; however, this length is theoretically possible for both the *sui* and *nian* years, though it is extremely rare. For instance, the *nian* year 1965 had a length of 353 days. The occurrence of years with a length of 385 days is also unlikely, with only two instances in the specified period, occurring in 2270 and 2289. It is important to note that the lengths of the *sui* and *nian* years do not align, even though the frequencies of their respective lengths are quite similar.

In common years, the duration is typically 354 days, while in embolismic years, it is typically 384 days.

The average length of common years during the considered period is 354.347 days, while that of embolismic years is 383.928 days. Additionally, the average length of the solar year over the three hundred years studied is 365.2558 days.

Years	Duration year *nian*	Duration year *sui*	Years	Duration year *nian*	Duration year *sui*
2024	354	355	2043	354	354
2025	384	384	2044	384	384
2026	354	354	2045	354	354
2027	354	354	2046	354	354
2028	384	384	2047	384	384
2029	355	354	2048	354	355
2030	354	355	2049	355	355
2031	384	384	2050	384	384
2032	355	354	2051	355	354
2033	384	355	2052	384	384
2034	354	384	2053	354	354
2035	354	354	2054	354	354
2036	384	384	2055	383	384
2037	354	354	2056	355	354
2038	354	355	2057	354	355
2039	384	384	2058	384	384
2040	355	355	2059	355	355
2041	355	354	2060	354	354
2042	384	384	2061	384	384

Table 2.14- **LENGTH OF *NIAN* AND *SUI* YEARS**. In the Chinese calendar, there are two types of lunisolar years distinguished by their starting times: the *nian* and *sui*. Both can be common years (consisting of 12 months) or embolismic years (with 13 months). The *nian* year begins on the first day of the first month, marking the New Year. In contrast, the *sui* year starts with the month following the 11th month, which is typically the 12th month or, in rare cases, the intercalary-11t month. The duration of the *nian* and *sui* years is not always the same, as illustrated in the accompanying table. Although most of the months of the *nian* and *sui* years overlap, the same does not happen with the months (one or, exceptionally, two) that come after the 11th month (month 12 and, exceptionally, the intercalary months-11 and intercalary-12). It may be that the 12th month of the *sui* year is of a different length from the 12th month of the *nian* year, which is why the length of both years may be different. It also happens when the embolismic month is the intercalary-11 or the intercalary-12, and the length of both years differs by one month.

353 days is theoretically possible, as demonstrated by the *nian* year 1965. From Table 1.14 we find that years of 385 days are unlikely.

According to Table 1.14, the most common lengths for lunisolar years are 354 days (six months with 30 days and six months with 29 days) and 384 days (comprising seven months of 30 days and six months of 29 days). Additionally, years with 355 days (seven months with 30 days and five months with 29 days) are also possible.

In Table 1.14, we observe that the lengths of the *sui* and *nian* years mostly align. However, in Table 2.14, which presents the years from 2024 to 2061, we find that in 12 cases, the lengths of the *nian* and *sui* years do not coincide. Additionally, there may be instances where the *sui* year is intercalary while the *nian* year is common, and vice versa.

Consecutive months of the same length

The lunar months in the calendar can only be 29 or 30 days long. Since the average length of lunations is more than 29 and a half days, there will generally be a nearly equal number of months of both lengths. However, this does not imply that the months with 29 and 30 days occur alternately, as in some computational calendars.

In Chapter 5, we studied lunar motion and observed that the length of lunations varies based on the positions of the Moon and the Sun in their elliptical orbits. As a result, we can anticipate periods with consecutive months of 30 days and periods with consecutive months of 29 days. Between 2025 and 2100, we will encounter two, three, and even four consecutive months with 30 days and two and three consecutive months of 29 days.

Table 3.14 presents the most notable scenarios: a sequence of four months with 30 days and a sequence of three months with 29 days. We observe that the months with 30 consecutive days occur in winter when the Sun is near its perihelion. During this time, the Sun's orbital movement is faster, requiring the Moon to cover a greater distance between successive conjunctions, resulting in longer lunations. In contrast, the succession of months with 29 days takes place in summer when the Sun is close to its aphelion.

Years	Consecutive months of 30 days
2036/2037	11, 12, 1, 2
2044/2045	10, 11, 12, 1
2052/2053	9, 10, 11, 12

Years	Consecutive months of 29 days
2089/2090	5, 6, 7
2098/2099	5, 6, 7

Table 3.14.- **CONSECUTIVE MONTHS OF 30 AND 29 DAYS.** The table on the left presents the three instances between 2025 and 2100 when there are four consecutive 30-day months. The table on the right shows the two instances during the same period when there are three consecutive 29-day months. While three consecutive 30-day months occur frequently, there are no instances of four consecutive 29-day months.

We find another interesting fact in Table 3.14: the groups of consecutive months of equal duration occur between 8 and 9 years, a value close to the 8.85 years of the lunar perigee period.

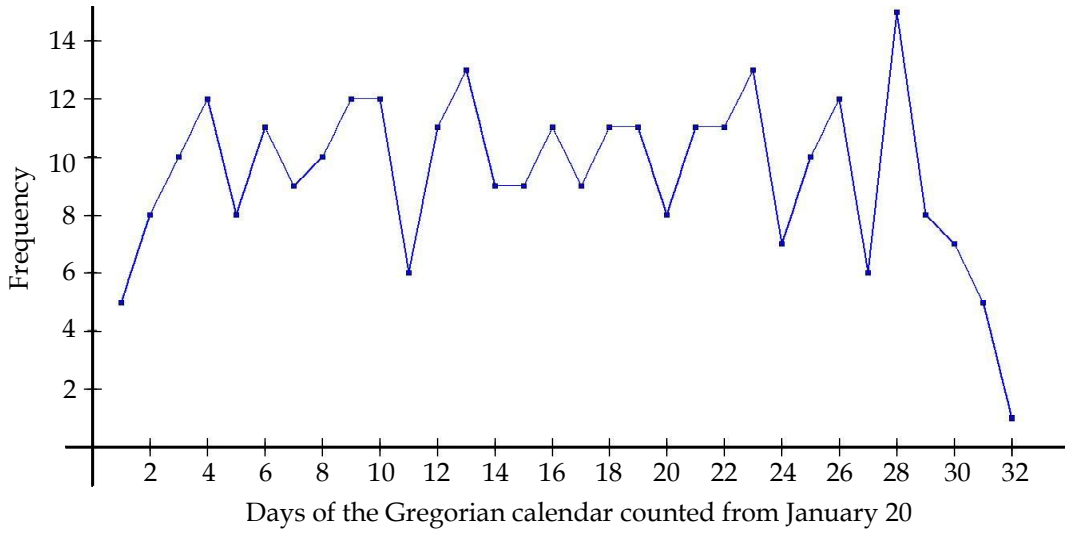
Gregorian date of Chinese New Year

The dates of a lunar calendar are not fixed relative to a solar calendar such as the Gregorian calendar. Lunar dates have limited mobility relative to the solar calendar, which is approximately one month long; this means that the Gregorian dates of Chinese New Year's day will have a certain mobility. In Table 4.14, we examined the Gregorian dates of Chinese New Year's day over three hundred years and found that they fall between January 21 and February 21, i.e. 32 days. Table 5.14 shows the

Gregorian date	Frequency	Gregorian date	Frequency
21 January	5	6 February	9
22 January	8	7 February	11
23 January	10	8 February	11
24 January	12	9 February	8
25 January	8	10 February	11
26 January	11	11 February	11
27 January	9	12 February	13
28 January	10	13 February	7
29 January	12	14 February	10
30 January	12	15 February	12
31 January	6	16 February	6
1 February	11	17 February	15
2 February	13	18 February	8
3 February	9	19 February	7
4 February	9	20 February	5
5 February	11	21 February	1

Table 4.14.- **POSSIBLE NEW YEAR DAYS**. The table presents the possible Gregorian dates for the Chinese New Year and their frequencies, covering 2024 to 2324. The New Year dates are quite varied, occurring between January 21 and February 21. The most common date for the New Year is February 17, with approximately a 5% probability. In contrast, February 21 is the least common date.

Year	Gregorian date of the New Year	Year	Gregorian date of the New Year	Year	Gregorian date of the New Year
2025	January 29	2051	Febraury 11	2077	Janaury 24
2026	February 17	2052	Febraury 1	2078	Febraury 12
2027	February 6	2053	Febraury 19	2079	Febraury 2
2028	January 26	2054	Febraury 8	2080	Janaury 22
2029	February 13	2055	Janaury 28	2081	Febraury 9
2030	February 3	2056	Febraury 15	2082	Janaury 29
2031	January 23	2057	Febraury 4	2083	Febraury 17
2032	February 11	2058	Janaury 24	2084	Febraury 6
2033	January 31	2059	Febraury 12	2085	Janaury 26
2034	February 19	2060	Febraury 2	2086	Febraury 14
2035	February 8	2062	Janaury 21	2087	Febraury 3
2036	January 28	2062	Febraury 9	2088	Janaury 24
2037	February 15	2063	Janaury 29	2089	Febraury 10
2038	February 4	2064	Febraury 17	2090	Janaury 30
2039	January 24	2065	Febraury 5	2091	Febraury 18
2040	February 12	2066	Janaury 26	2092	Febraury 7
2041	February 1	2067	Febraury 14	2093	Janaury 27
2042	Janaury 22	2068	Febraury 3	2094	Febraury 15
2043	February 10	2069	Janaury 23	2095	Febraury 5
2044	Janaury 30	2070	Febraury 11	2096	Janaury 25
2045	February 17	2071	Janaury 31	2097	Febraury 12
2046	February 6	2072	Febraury 19	2098	Febraury 1
2047	January 26	2073	Febraury 7	2099	Janaury 21
2048	February 14	2074	Janaury 27	2100	Febraury 9
2049	Febraury 2	2075	Febraury 15	2101	Janaury 29
2050	Janaury 23	2076	Febraury 5	2102	Febraury 17



Ilustration 1.14.- **FREQUENCY OF NEW YEAR'S DAYS**. The graph represents the frequency of New Year's days between the years 2024 and 2342, taken from Table 4.14. We find no pattern in the Gregorian date of the New Year's day, although the two extreme days (January 21 and February 21) are the least probable.

New Year's days from 2025 to 2102.

Not all potential dates for the New Year are equally probable. Table 4.14 presents the frequencies over three hundred years, revealing a significant variation in dates. Within this period, the most likely dates for the New Year are February 2, February 17, and February 12, while the least likely dates are February 21 and January 21. Additionally, the February Moon marks the beginning of the year in 65.8% of the cases examined, whereas only 34.2% correspond to the January Moon. In Illustration 1.14, we provide a graphical representation of the dispersion of the Gregorian dates of the Chinese New Year.

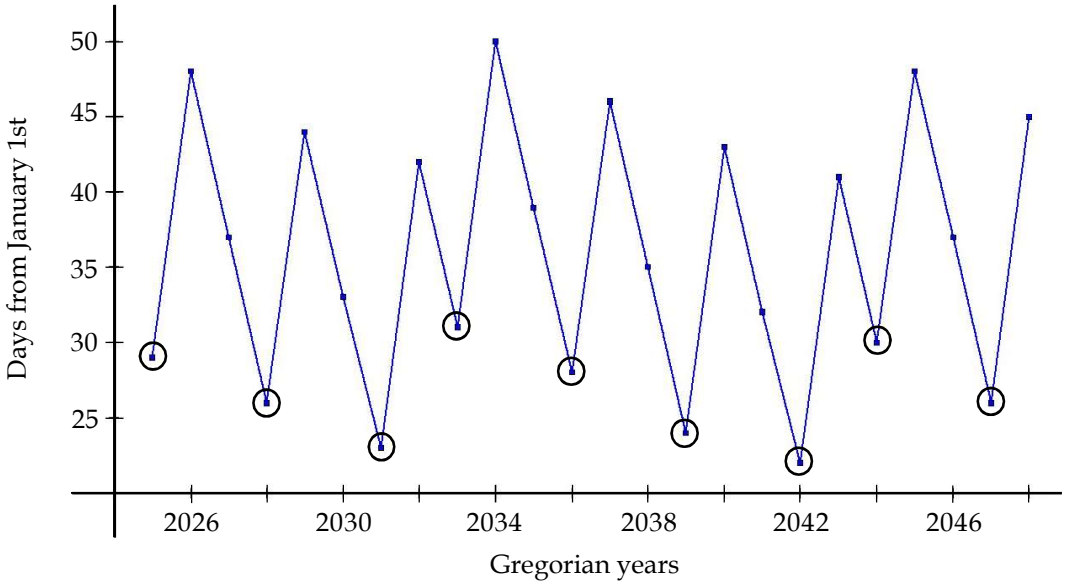
Chinese New Year's Day Gregorian Date

When there is a common year, which is 11 days shorter than a solar year, the following New Year's day will come 11 days earlier. Whereas when there is an abundant year, which is 19 days longer than a solar year, New Year's day will come 19 days later.

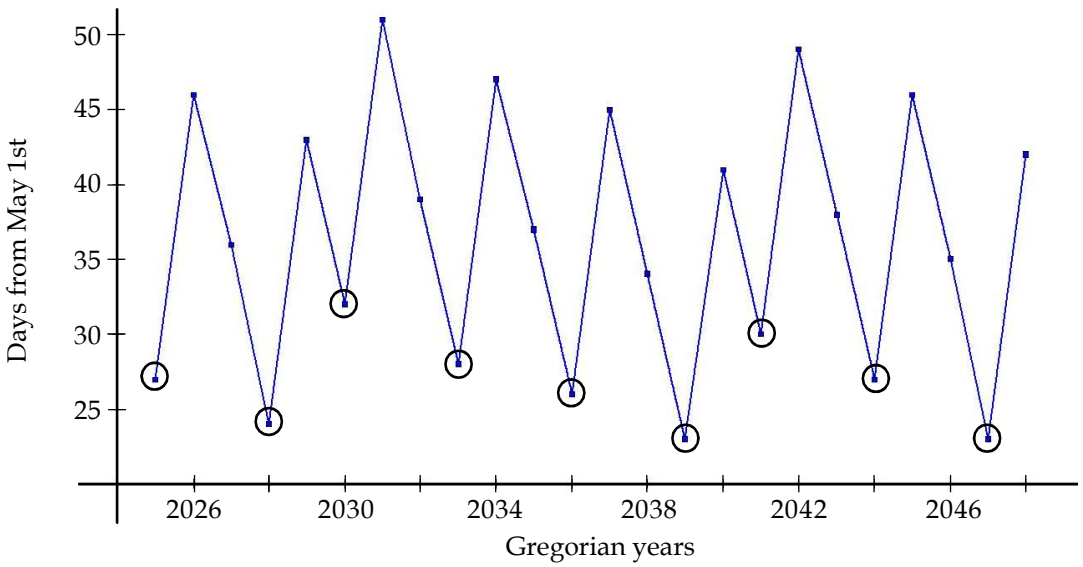
In Illustration 2.14, we show the mobility of the Chinese New Year, checking the delays and advances we have previously indicated. We see that after an intercalary year (marked with a circle), the beginning of the Chinese year is delayed and advanced after a common year.

Table 5.14 presents the numerical delays and advances of the New Year's date from one year to the next. Since the lengths of both common and intercalary years can vary, the days of advance are not always 11, nor are the days of delay consis-

Table 5.14.- **NEW YEAR DAYS**. In the table on the previous page, we calculated the Chinese New Year dates from 2025 to 2102. The Chinese lunisolar calendar is more likely to begin in February than January.



Ilustration 2.14.- **CHINESE NEW YEAR DATE MOBILITY.** The vertical axis represents the number of days since the start of the Gregorian year. The points on the graph indicate the dates of the Chinese New Year for the corresponding year marked on the horizontal axis. Intercalary years are highlighted in circles. After a common year, Chinese New Year occurs approximately 11 days earlier than in the previous year. However, after a intercalary year, it falls approximately 19 days later than the previous year, helping to stabilize the date of the first day of the new year.



Ilustration 3.14.- **MOBILITY OF THE BEGINNING OF THE FIFTH MONTH OF THE LUNISOLAR YEAR.** The shift of the beginning of all lunar months relative to the Gregorian calendar follows the same pattern as the shift of the date of New Year's day. In the months of years preceding an intercalary year, the Gregorian date of the beginning of the fifth month is earlier, and in the fifth months of years following an intercalary year, the Gregorian dates will be later. Circles represent intercalary years.

Gregorian date of the New Year	Days between two New Year's dates	Length of the solar year	Length of the lunisolar year
2025/01/29			
	+19	365	384
2026/02/17			
	-11	365	354
2027/02/06			
	-11	365	354
2028/01/26			
	+18	366	384
2029/02/13			
	-10	365	355
2030/02/03			
	-11	365	354
2031/01/23			
	+19	365	384
2032/02/11			
	-11	366	355
2033/01/31			
	+19	365	384
2034/02/19			
	-11	365	354
2035/02/08			
	-11	365	354
2036/01/28			
	+18	366	384
2037/02/15			
	-11	365	354
2038/02/04			
	-11	365	354
2039/01/24			
	+19	365	384
2040/02/12			
	-11	366	355
2042/02/01			
	-10	365	355
2043/01/22			
	+19	365	384
2043/02/10			
	-11	365	354
2044/01/30			
	+18	366	384
2045/02/17			
	-11	365	354
2046/02/06			
	-11	365	354
2047/01/26			
	+19	365	384
2048/02/14			

tently 19; some variability is involved. The days of advance or delay are the difference between the length of the lunisolar year and the solar year, as seen in Table 5.14.

This mobility of the first day of the year occurs with any day of the lunisolar year, in particular, with the beginning of any other month. In Illustration 3.14, we see the mobility of the first day of month 5, where we again observe the same rhythm of advances and delays.

Chinese calendar festival

Among the festivals associated with the Chinese calendar, we differentiate those established by the solar calendar (*Qingming* and *Dongzhi*) from the others that are based on the lunar calendar.

* *New Year:*

The New Year is the most significant festival in the Chinese calendar. It is called the Spring Festival and is celebrated on the first day of the first month. The Lantern Festival marks the conclusion of the Chinese New Year festivities and is observed on the 15th day of the first lunar month.

The New Year is a public holiday celebrated in China, Taiwan, Hong Kong, Macau, Singapore, South Korea, Malaysia, Laos, Vietnam, and Mauritius, as well as by Chinese communities worldwide.

* *Quingming:*

Qingming, also known as Tomb-Sweeping Day, is a time to honour and celebrate life while commemorating those who have passed away. It is observed during the minor solar term J3, which occurs between April 4 and 6 in the Gregorian calendar. It is a public holiday in Hong Kong, Macau, Taiwan and South Korea.

* *Dragon Boat:*

The Dragon Boat Festival, also known as *Duanwu*, is celebrated on the fifth day of the fifth month of the lunar calendar, which is why it is often referred to as the Double-Fifth Festival. This traditional festival combines various customs and is recognized as a public holiday in China, Hong Kong, Macau, Taiwan, and South Korea. It is also celebrated in several other Asian countries.

* *Mid-Autumn:*

Mid-Autumn Festival (*Zhongqiu*) or harvest festival, is celebrated on the 15th day of the eighth month. It is a public holiday in Hong Kong, Macau, Taiwan and South Korea.

Table 5.14.- **CHINESE NEW YEAR DATE MOBILITY.** We examine how the date of the first day of the Chinese lunisolar year moves in relation to the Gregorian calendar. In a common year, New Year's day falls 11 or 10 days earlier than the previous year. Conversely, in the years following an intercalary year, New Year's day occurs 18 or 19 days later than the previous year. The variation in New Year's day is attributed to the difference between the lengths of the lunisolar year and the solar year, as outlined in the table's fourth and third columns. The mobility of the start of the lunisolar year primarily results from the differing lengths of common and embolismic years. Some years shift the date earlier due to being shorter than the solar year, while others push the date later because they are longer than the solar year.

* *Double Ninth:*
The Double Ninth Festival (*Chongyang*) is a public holiday in Hong Kong and Macau. Celebrated on the ninth day of the ninth month, it is associated with the concepts of yin and yang.
* *Dongzhi:*
Dongzhi is the winter solstice, which can be between December 21 and 23. It is a public holiday in Macau.

Table 6.14 shows the Gregorian calendar dates of the leading traditional Chinese festivals. *Qingming* and *Dongzhi* are solar festivals and, therefore, occur on approximately the same dates, which is not the case for the other festivals related to the lunar calendar.

New Year	*Qingming*	Dragon Boat	Mid-Autum	Double Ninth	*Dongzhi*
2025/01/29	2025/04/04	2025/05/31	2025/10/06	2025/10/29	2025/12/21
2026/02/17	2026/04/05	2026/06/19	2026/09/25	2026/10/18	2026/12/22
2027/02/06	2027/04/05	2027/06/09	2027/09/15	2027/10(08	2027/12/22
2028/01/24	2028/04/04	2028/05/28	2028/10/03	2028/10/26	2028/12/21
2029/02/13	2029/04/04	2029/06/16	2029/09/22	2029/10/16	2029/12/21
2030/02/03	2030/04/05	2030/06/05	2030/09/12	2030/10/05	2030/12/22
2031/01/23	2031/04/05	2031/06/24	2031/10/01	2031/10/24	2031/12/22
2032/02/11	2032/04/04	2032/06/12	2032/09/19	2032/10/10	2032/12/21
2033/01/31	2033/04/04	2033/06/01	2033/09/08	2033/10/01	2033/12/21
2034/02/19	2034/04/05	2034/06/20	2034/09/27	2034/10/20	2034/12/22
2035/02/08	2035/04/05	2035/06/10	2035/09/16	2035/10/09	2035/12/22
2036/01/28	2036/04/04	2036/05/30	2036/10/04	2036/10/27	2036/12/21
2037/02/15	2037/04/04	2037/06/18	2037/09/24	2037/10/17	2037/12/21
2038/02/04	2038/04/05	2038/06/07	2038/09/13	2038/10/07	2038/12/22
2039/01/24	2039/04/05	2039/05/27	2039/10/02	2039/10/26	2039/12/22
2040/02/12	2040/04/04	2040/06/14	2040/09/20	2040/10/14	2040/12/21

Table 6.14.- **GREGORIAN DATES OF THE MAIN FESTIVALS OF THE CHINESE CALENDAR.** The table lists the Gregorian calendar dates for the six main traditional Chinese festivals up to 2040. It is important to note that *Qingming* and *Dongzhi* occur on similar Gregorian dates because these festivals are linked to the Chinese solar calendar: *Qingming* coincides with the minor solar term J1, while D*ongzhi* aligns with the major solar term Z12. The other festivals are based on lunar months, which means their dates can shift by approximately one month when expressed in Gregorian calendar terms.

Wrong rules

Several misconceptions exist about determining the Chinese New Year and preparing the calendar. Although the following rules are not accurate, they generally hold.

* *The New Year is the second month after the month of the winter solstice*. This is what happens in the vast majority of cases. However, if the intercalary month is the 11th or the 12th, there are three months between the 11th and 1st months; this rarely happens. As Table 1.12 shows, in six hundred years, it will only happen in 5 years.
* *The New Year coincides with the new Moon closest to the minor solar term* J1. The solar term J1 can occur between the 3rd, 4th and 5th of February and the latest date of the New Year is the 21st of February. So if New Year's day is very late, J1 is closer to the new Moon before the New Year's Moon. This erroneous rule is why the New Year is called the spring festival because it is often closer to the term *Lichum* or the beginning of spring.
* *The New Year falls on the day of the first new Moon after the solar term Z12*. The solar term Z12 occurs on the 19th, 20th and 21st of January. If the New Year is very late, i.e., close to the 21st of February, then there may be enough days between Z12 and the New Year to fit a month; therefore, the above rule will be violated.
* *The intercalary month is the empty month or one without a major solar term of the nian year*. Intercalary months are determined by the *sui* year, not by the *nian* year. The *sui* intercalary year usually corresponds to the *nian* intercalary year. However, if the intercalary month is the 11th or the 12th, this correspondence does not occur, and the previous rule is broken since the intercalary months 11 and 12 are not shared by the *sui* and the *nian* years. This situation will occur in 2034; according to the previous rule, the intercalary month would be the eighth month of 2033. However, this month corresponds to a non-intercalary *sui* year and, therefore, is a fake month.
* *There are no intercalary months 11 and 12.* Although unlikely, intercalary month-11 and intercalary-12 may exist. Table 1.11 shows that the astronomical solar months 11 and 12 lengths are 29.44 and 29.59 days. Therefore, the calendar solar months can have 29 and 30 days, while the lunar months can be 29 and 30 days. From Illustration 4.11, we find that a solar month of 30 days could correspond to an empty lunar month and, therefore, intercalary if the lunar month had 29 days, which can occur, although it is very unlikely.
* *The number of lunar months coincides with the number of major solar terms*. This rule is exceptionally violated when there is a fake or double lunar month.

Chinese calendar with average values of the lunation and the winter seasonal year

The current Chinese calendar is based on true new Moons and winter solstices, However, it is also possible to create an alternative calendar that adheres to all the existing rules while using average values for the lunar cycle and the winter seasonal year.

To construct the calendar, we need the Gregorian reference dates for the

Month		Start date of the month		Date of major solar terms		
M	T	Mean	True	Mean	True	
11	11	2024/12/01	2024/12/01	2024/12/21	2024/12/21	Z11
12	12	2024/12/31	2024/12/31	2025/01/21	2025/01/20	Z12
1	1	2025/01/29	2025/01/29	2025/02/20	2025/02/18	Z1
2	2	2025/02/28	2025/02/28	2025/03/23	2025/03/20	Z2
3	3	2025/03/29	2025/03/29	2025/04/22	2025/04/20	Z3
4	4	2025/04/28	2025/04/28	2025/05/22	2025/05/21	Z4
5	5	2025/05/27	2025/05/27	2025/06/22	2025/06/21	Z5
6	6	2025/06/26	2025/06/25	2025/07/22	2025/07/22	Z6
7	6i	2025/07/25	2025/07/25	2025/08/22	---	Z7/--
8	7	2025/08/24	2025/08/23	2025/09/21	2025/08/23	Z8/Z7
8i	8	2025/09/22	2025/09/22	----	2025/09/23	--/Z8
9	9	2025/10/22	2025/10/21	2025/10/22	2025/10/23	Z9
10	10	2025/11/20	2025/11/20	2025/11/21	2025/11/22	Z10
11	11	2025/12/20	2025/12/20	2025/12/21	2025/12/21	Z11
12	12	2026/01/19	2026/01/19	2026/01/21	2026/01/20	Z12
1	1	2026/02/17	2026/02/17	2026/02/20	2026/02/18	Z1
2	2	2026/03/19	2026/03/19	2026/03/23	2026/03/20	Z2
3	3	2026/04/17	2026/04/17	2026/04/22	2026/04/20	Z3
4	4	2026/05/17	2026/05/17	2026/05/23	2026/05/21	Z4
5	5	2026/06/15	2026/06/15	2026/06/22	2026/06/21	Z5
6	6	2026/07/15	2026/07/14	2026/07/23	2026/07/23	Z6
7	7	2026/08/13	2026/08/13	2026/08/22	2026/08/23	Z7
8	8	2026/09/12	2026/09/11	2026/09/21	2026/09/23	Z8
9	9	2026/10/11	2026/10/10	2026/10/22	2026/10/23	Z9
10	10	2026/11/10	2026/11/09	2026/11/21	2026/11/22	Z10
11	11	2026/12/09	2026/12/09	2026/12/22	2026/12/22	Z11
12	12	2027/01/08	2027/01/08	2027/01/21	2027/01/20	Z12
1	1	2027/02/06	2027/02/06	2027/02/21	2027/02/19	Z1

lunation and the solar year and their mean values. For example, we start with the lunation on December 1, 2024, corresponding to Julian Day 2,460,646.09821. We also consider the beginning of the solar term count, precisely the major term Z11, which occurs on December 21, 2024, and is associated with Julian Day 2,460,666.22253. The mean values we will use are 29.530589 days for the lunation and 30.436854 days for the solar month.

In Table 7.14, the calendar for the years 2025/2026 is obtained from the mean values of the lunation and the year, and we compare them with the calendar obtained from the true astronomical values. We find that there are some differences in the beginning of the lunar months, and we also find that there is no coincidence between the position of the intercalary month of the year 2025, while in one calen-

Year	Chinese date of January 1st	Year	Chinese date of January 1st
2025	2024/12/02	2038	2037/11/26
2026	2025/11/13	2039	2038/12/07
2027	2026/11/24	2040	2039/11/17
2028	2027/12/05	2041	2040/11/29
2029	2028/11/17	2042	2041/12/10
2030	2029/11/28	2043	2042/11/21
2031	2030/12/08	2044	2043/12/02
2032	2031/11/19	2045	2044/11/14
2033	2032/12/01	2046	2045/11/25
2034	2033/11i/11	2047	2046/12/06
2035	2034/11/22	2048	2047/11/16
2036	2035/12/04	2049	2048/11/28
2037	2036/11/16	2050	2049/12/08

Table 8.14.- **CHINESE DATE OF JANUARY 1 GREGORIAN.** We show the Chinese calendar date of the first day of the Gregorian year, January 1. We check that January 1 can only fall in the 11th or 12th month or exceptionally in the intercalary-11 month. We check that the Chinese day of January 1 is in the previous *nian* year since it is always in the 11th or 12th month of the previous year.

Table 7.14.- **CHINESE CALENDAR WITH AVERAGE VALUES.** In the table on the previous page, we have reconstructed the Chinese calendar for the years 2025/2026, using average values for lunation and the winter seasonal year. We find some differences with respect to the Chinese calendar constructed with true astronomical values. The differences are greater in the solar terms than in the new Moons. We also find that the intercalary month is different in both calendars.

New Moon							
True		Mean			First crescent		
Date	Time	Date	Time	Difference	Date	Time	Difference
01/29	20:36	01/29	09:57	-10.7	01/31	18:29	45.9
02/28	08:45	02/28	02:40	-6.1	03/01	18:02	33.3
03/29	18:58	03/29	11:24	-7.6	03/30	18:40	24.1
04/28	03:31	04/28	00:08	-3.4	04/29	19:02	39.5
05/27	11:02	05/27	12:52	1.8	05/28	19:29	32.5
06/25	18:32	06/26	01:37	7.1	06/26	19:47	25.3
07/25	03:11	07/25	14:20	11.2	07/26	19:29	40.3
08/23	14:06	08/24	03:05	13.0	08/25	18:52	52.8
09/22	03:54	09/22	15:48	11.9	09/24	18:04	62.2
10/21	20:25	10/22	04:33	8.1	10/24	17:18	68.9
11/20	14:47	11/20	17:17	2.5	11/22	16:49	50.03
12/20	09:43	12/20	06:00	-3.7	12/22	16:48	55.1

Table 9.14.- **TRUE AND MEAN NEW MOONS, AND FIRST CRESCENT OBSERVATION.** We compared the times of the true new Moon and the mean new Moon for 2025 and found a periodic difference of up to 13 hours. When we compared the true new Moon with the observation of the first crescent in Beijing, we noted a difference ranging from one day to about three days. It is important to mention that the observation of the crescent occurs shortly after sunset. To calculate the visibility of the first crescent, we considered a geographical position of 39º 54′ 15′′ N latitude and 116º 24′ 27′′ E longitude, with an altitude of 44 meters, relative humidity of 25%, and a sighting probability of 50 %. We used the software «Crescent Moon Visibility» for our calculations.

dar, it is month 8i, in the other, it is month 6i. We also find that the differences between the mean and true new Moon dates are smaller than the differences between the solar terms of one and the other type.

Gregorian New Year on the Chinese calendar

We have previously seen that the Gregorian dates of the Chinese New Year can be from January 21 to February 21, both days included. Now we wonder which Chinese dates the new year of the Gregorian calendar can occur, that is, on January 1.

The latest date on the Chinese calendar for January 1st is the 11th day of the 12th month. If the 12th month had 30 days, the first day of the first month would coincide with January 21st, the earliest date of the New Year.

For the same reasons as above, the earliest Chinese date on which January 1 can occur is the 7th of the 11th month (or exceptionally, the intercalary month-11 or

the 12th month if there is an intercalary-12). In Table 8.14, we put the Chinese dates of Gregorian January 1 from 2025 to 2050.

True and mean new Moons and first observation of the crescent

In the Chinese calendar, the month begins with the day when, according to the Beijing time zone, the true conjunction of the Sun and the Moon occurs, meaning that their apparent ecliptic longitudes are equal. However, there are other ways of defining the new Moon.

The mean new Moon occurs when the mean longitudes of the Sun and the Moon (L_m and L'_m , Chapter 5) are equal. An alternative definition of the new Moon is when the lunar crescent is observed after it has been in conjunction with the Sun.

Another possible definition is the topocentric new Moon, which we define as the moment when the topocentric arc-light, or topocentric angle between the centres of the Sun and the Moon, is minimal.

Table 9.14 compares the true new Moons with their mean values. The differ-

Date	Topocentric New Moon		Geocentric New Moon		Difference of time	Difference of arc-ligth
	Hour	Arc-light	Hour	Arc-light		
01/13	03:08	03-11	05:01	02-52	- 01:53	19′
02/11	20:37	04-47	19:07	04-36	01:30	11′
03/13	14:37	05-32	10:22	04-57	04:15	35′
04/12	01:57	04-32	02:32	03-53	- 00:35	39′
05/11	20:28	02-07	19:01	01-45	01:27	22′
06/10	09:29	00-29	10:54	00-50	- 01:25	-21′
07/10	00:32	02-22	01:18	03-01	- 00:46	- 48′
08/08	16:13	04-14	13:51	04-40	02:22	- 26′
09/06	23:52	04-17	24:53	04.57	- 01:01	- 40′
10/06	12:40	03-25	11:07	03-53	01:33	36′
11/04	21:44	01-10	21:16	01-43	00:28	- 33′
12/04	05:47	01-17	07:44	00-59	-01:57	18′

Table 10.14.- **GEOCENTRIC AND TOPOCENTRIC NEW MOONS.** The table shows the geocentric and topocentric new Moons of the year 2021 for the latitude of 36º and in the civil time of the prime meridian. We find a difference between both new Moons that exceeds four hours. We also find differences between the geocentric and topocentric arc-ligth at the time of the corresponding new Moons. The time of the new Moons is hour:minute, and the arc-ligth is degree-minute.

ences can reach up to 13 hours in 2025, showing a periodic variation. Additionally, the table compares the true new Moons with crescent observations made in Beijing. We observe that these differences can be as much as three days. It is important to note that the crescent is first seen shortly after sunset.

In Table 10.14, we calculate the moments of the occurrence of the geocentric and topocentric new Moons for the year 2021. The differences can exceed four hours. If the topocentric arc-light is less than the sum of the Sun and Moon radii, we will have a solar eclipse. The moment of the maximum eclipse corresponds to the topocentric new Moon, as we have defined it in terms of the arc-light. Note that the maximum eclipse does not correspond to the geocentric new Moon.

Usually, a solar eclipse is observed on the day of the geocentric new Moon. However, in very exceptional circumstances, the solar eclipse may occur entirely on a day other than the geocentric new Moon. In the Chinese calendar, solar eclipses usually occur on the first day of the lunar month.

15.- A brief history of the Chinese calendar

The evolution of the Chinese calendar has been different from that of the main calendars, which are adjusted to the average values of the Moon and the Sun. The trend of the Chinese calendar has been a gradual search for astronomical accuracy until today, where only a precision of one second or less is acceptable in calculating the new Moons and winter solstices, astronomical phenomena that regulate the Chinese calendar. The reason for this trend must be sought in the philosophical foundation of the power of the ancient emperors.

The emperor's legitimacy came from the philosophical-political concept of the Mandate of Heaven, which favoured just rulers and punished, leaving unprotected, those who misused power, which would facilitate the loss of the emperor's authority and the advent of a new ruler.

The emperor based his authority on being the Son of Heaven. He was, therefore, obliged to ensure that his government published a calendar in harmony with nature, which forced calendarist reforms to tend towards astronomical accuracy. These political and philosophical reasons explain why the calendar was important to the State and why the services of Indian, Muslim, and, finally, Christian astronomers were not hesitant to be used, to the detriment of local astronomers.

In comparison, the Western calendar evolved into a chronological system that did not strictly adhere to astronomical laws but instead sought to fix past and future events in the calendar.

While the Western calendar allows a historical event to be precisely fixed on a time scale, the Chinese calendar does not have this chronological capacity, to the point that it is even devoid of an era, since, as we say, the most important thing was to make the calendar compatible with astronomical movements as accurately as possible.

As we have already indicated, while the Western calendar was intended to be popular, in the sense of not being subject to any authority and therefore manageable by the population, the Chinese calendar depended and continues to depend on scientific and political authority. It became a system that could only be managed by specialists and whose publication was the responsibility of the emperor.

According to some authors, the Chinese calendar has registered a hundred alterations in this incessant evolution towards astronomical perfection. These modifications must be divided into true calendarist reforms, of which we distinguish five main ones, and punctual alterations of various kinds, among which are modi-

fications in the length of the months, alterations in the beginning of the lunar year, changes in astronomical theory, or changes in the time scale.

The concordance between the calendar and astronomy is not readily observable by the population. Unless astronomical measurements are made, it is not noticeable that the winter solstice occurs on its astronomical date or a few days before or after. The date of the new Moon can be displaced one or two days from the astronomical date without it being perceptible by the population. The new Moon is not an observable phenomenon that allows us to verify a lag between the calendar and astronomy. Nor is it useful to observe the full Moon, which, as we have seen, can occur on the 14th, 15th, 16th or 17th of the lunar month, a dispersion that would mask the possible astronomical error. The situation changes if the errors committed in determining the new Moons and the winter solstices are cumulative because then the moment would come when the accumulated error would reach such a magnitude that a calendar malfunction would be apparent.

Only a solar eclipse could warn the population that the calendar was not in step with astronomy. Solar eclipses are topocentric phenomena greatly affected by the geographic position of the observer. Usually, the solar eclipse occurs on the day of the new Moon, that is, on the first day of the lunar month, a coincidence that is noticeable by the population *, who could thus judge the adequacy of the calendar.

The evolution of the Chinese calendar has had five phases, which we will detail later: observational, computational, astronomical with the mean motion of the Moon and the Sun, astronomical with the true motion of the Moon and finally, astronomical using true motions of both the Moon and the Sun, evolution where we can see, as we have said, the continued tendency towards astronomical accuracy.

We note that for calendar purposes, the accuracy of the calculation of the moment of the new Moon is more important than the precise determination of the winter solstice. If the new Moon is close to midnight, a small error in its determination can cause the first day of the lunar month to be different; that is, a lunar month can be short or long. However, if, due to a calculation error, a solar term is a day or another because it is close to midnight, it has no consequence in the calendar unless it is also close to the new Moon, but these two coincidences are exceptional.

From the above, we understand that Chinese astronomers used the true motions of the Moon together with the mean motions of the Sun for more than a thousand years. This situation changed in the mid-17th century when Western astronomical knowledge reached China.

First period

Some authors trace the origin of the Chinese calendar back to the 13th century BCE and maintain the belief that it was observational, meaning that the month began

*The new Moon is defined relative to the Earth's centre in the current calendar. However, solar eclipses are local phenomena where the observation is made on the surface of the Earth and not at its centre, thus having to consider the phenomenon of lunar parallax. That is, the solar eclipse is related to what we call the topocentric new Moon in Chapter 14. With current methods, it may be unusual that the solar eclipse is not on the first day of the lunar month. Furthermore, given that the geographical position is decisive, the solar eclipse is observable in one place, not another.

when the Moon was first visible on the western horizon shortly after sunset after having been in conjunction with the Sun and, therefore, invisible. In this primitive calendar, there must have been some mechanism to determine when a year was abundant or thirteen months long. It is possible to speculate that, as with the Jews, it was the weather conditions and the state of the vegetation that determined when to place the embolismic month. This unsafe intercalation method must have caused years with an excessive length of 14 months.

We have fragmentary information from a calendar used in the historical period Spring and Autumn (722 BCE—481 BCE), which does not allow us to state with certainty how the intercalary month was placed, although the winter solstice was already determined by observation at that time. In this early period, the motion of the Sun did not define solar terms.

When discussing the earliest stages of the Chinese calendar, it should be noted that China was divided into many states, each with its calendar. Although similar, they differed in some aspects, such as the month the year began or the reference dates for calculating the new Moons and winter solstices.

We have reliable records of Chinese calendar dates from 841 BCE, based on the *Records of the Grand Historian* or *Shiji*, which Sima Qian began writing in the 2nd century BCE during the Han Dynasty.

After 589 BCE, 7 intercalary months were added every 19 years, indicating the use of the Meton cycle and a computational calendar. The method used at that time did not specify the exact timing of the intercalary month; it was always placed at the end of the year.

The first major calendar reform took place in 104 BCE, taking on some of its defining characteristics today. The no *zhongqi* rule was introduced, meaning that the intercalary month did not have a major solar term, and the 11th month contained the winter solstice. Another change recorded in this reform was that the beginning of the year was moved from month 10 to month 1, so the year 104 BC had 15 months.

The calendar before the Jesuit reform

From 104 BC until the year before the last major calendar reform in 1645, over forty different calendar systems were used. During the periods when China was divided into several states, each followed its own calendar; although similar, they were not the same. Added to this were frequent changes in the lunar month with which the year began. During the Xin dynasty in 9 CE, month 12 was designated the year's first month. After the overthrow of this dynasty and the rise to power of the Han dynasty, month 1 returned to being the first month of the year. In 237 CE, month 12 returned to being the first month of the year, but the change was reversed when the next emperor ascended to the throne. Between 689 and 701, the year began in month 11. Finally, in 761, month 11 returned to being the first month of the year, but the old system was restored the following year.

In the middle of the 5th century, the use of the true Moon to prepare the calendar was proposed, but with this method, several consecutive months of the same length could occur, which did not occur in the calendar that used the mean Moon, so this method was not adopted.

In 619 CE, during the Tang Dynasty, a significant calendar reform was implemented. This reform focused on using the true motion of the Moon (*dingshou*)

for calculations, rather than the mean motion (*pingshou*) that had been used previously. The mean motion of the Sun (*pingqi*) continued to be utilized for calculations as well. However, it became apparent that this new method resulted in several consecutive months of the same length, leading to a return to the old system based on mean Moons in 645 CE. Ultimately, in the calendar of 665 CE, the true Moon was definitively reinstated.

When the mean motions of the Moon and Sun were used, all months were equivalent and could be intercalary with equal probability. There was no possibility of fake months, that is, months without a major solar term but not intercalary. When using the true Moons (*dingshou*), lunations have variable durations (Chapter 5), reaching shorter lengths when the Sun is at aphelion during the summer. This circumstance makes the summer months more likely to be intercalary. So, with the *dingshou* system, not all months have the same probability of being intercalary.

From the mean Sun to the true Sun

The Chinese lunisolar calendar acquired its current rules in the mid-17th century when Jesuit missionaries arrived in China. They brought Western astronomical knowledge, which made it possible to prepare a more accurate calendar closer to the movement of the Moon and the Sun.

In 1582 the first Jesuit missionary, the Italian Matteo Ricci, arrived in China. A significant event in our history occurred when the Jesuit converted a high-ranking official named Xu Gangqi to Christianity, who collaborated with Ricci to translate Western scientific works.

In 1629, Emperor Chongzhen, the last of the Ming Dynasty, appointed Xu to organize a committee to improve the calendar. To evaluate the different calculation methods, Xu asked Chinese astronomers, Muslims, and the Jesuits to predict an upcoming solar eclipse. Xu was impressed by the excellent results of the Jesuit missionaries and thus adhered to their astronomical theory.

As a result of the superiority of Western astronomy, Xu appointed the Swiss Johannes Schreck and the Italian Niccolò Longbardo as members of the calendar bureau. In addition, shortly before his death, Xu began writing a book on the calendar based on Western astronomy called *Chongzhen Lishu* (Series on Calendar Science in the Chongzhen Kingdom) which was completed by Li Tianjing in 1635.

Following Schreck's death in 1630, he was replaced in the Calendar Office by German Adam Schall von Bell, the first Jesuit Royal Astronomer in China. He was specifically charged with developing a new calendar based on Western astronomy.

At the beginning of the Qing Dynasty in 1644, Adam Schall used the *Chongshen Lishu* as the basis for the new calendar, preparing the work *Xiyang Xinfa Lishu* (New Western Methods for Calculating the Calendar). Until 1629, Jesuit astronomers used Ptolemy's theory to make their predictions, but Schall based his new method on Tycho de Brahe's theory, with which he prepared the new calendar for 1645.

The annual calendar based on the new method was known as the *Shixian Shu* (Book of the Establishment of Time), and its main contents were the length of lunar months in a year, the date and time of each solar term, the dates and times of the phases of the Moon (new, first quarter, full and last quarter); the time of sunrise and sunset and the length of day and night for certain principal days, all calculated for Beijing local time; it also contained a summary of astrological predictions, with

favourable and unfavourable days.

The triumph of Western astronomy, which ended with its application to the calendar, was favored by the decline of astronomical knowledge during the Ming Dynasty.

The main feature of the new calendar was the abandonment of the *pinqi* rule and the use of the true values of solar motion or *dingqi* rule, thus fully establishing the calendar as we know it today.

In the quest for greater astronomical accuracy, the new calendar gave rise to a complication that has confused until recently. With the new method, solar months have variable lengths ranging from 29 to 32 days (Chapter 11). It is then possible (as we have already pointed out) for there to be double lunar months, that is, months with two major solar terms, but since there must be twelve such terms in each period of a solar year, there must be an empty lunar month, that is, without any major solar term, which is not embolismic.

According to the *pinqi* rule, an intercalary month is empty, and all empty months must be intercalary. However, according to the *dingqi* rule, there are non-intercalary empty months, which we have called fake months. Some intercalary years have two empty months. It is therefore necessary to establish a new rule to determine which of these two months is the embolism. With the new calendar, the no *zhongqi* rule had to be supplemented with the rule that the intercalary month is the first empty month of the *sui* year. We have already discussed that applying the previous rule to the *nian* year is a mistake that has occurred on some occasions. It should be noted that until 1990, all Chinese calendars had the intercalary month of 2033 incorrectly placed due to a misunderstanding of the no *zhongqi* rule.

Another circumstance with the new calendar is the frequency of intercalary months, which differs significantly from the calendar that used the *pingqi* rule. The frequency was already altered when using the pingshou rule, with the true Sun, the frequencies of the months intercalary are very different from each other (Table 1.12).

When the mean motion of the Sun was used in the calendar, the numbering of the lunar and solar months coincided. Thus, in lunar month 1, there was the solar term Z1, and the same equivalence was maintained in all the remaining months. Note that the intercalary month has no numbering, and neither does it have a major solar term; therefore, they do not affect the previous equivalence.

As we have explained on another occasion, any astronomical calendar, particularly the Chinese one, must refer time to a meridian. Throughout the Qing dynasty and until 1924, the time in calculating astronomical phenomena was the apparent solar time of Beijing *. The circumference was divided into 365^{o} and a quarter to facilitate the astronomical calculations. This system was in use until the reform of the Jesuits, who also introduced the sexagesimal system in the measurement of angles, introducing minutes and seconds of arc, which replaced the decimal system traditionally used by the Chinese.

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* True or apparent solar time is the hour angle of the centre of the true Sun increased by 12 hours. The true solar day is then the time elapsed between two passages of the true Sun's centre through the location's upper meridian. Mean solar time is obtained from true time using the equation of time.

The Case of the Calendar

The *Shixian* system established by the Jesuits in 1645 was astronomically superior to the *Datong Li* (Great Unity Calendar) system used in the late Ming Dynasty and adopted in 1383 and was almost identical to the *Shoushi Li* (Season Granting Calendar) devised in 1280.

However, problems arose in the application of the new system, where not only technical astronomical considerations intervened but also political and even religious problems, which finally led to what has been called the Calendar Case.

The main character in this story was the Chinese official Yang Guangxian, who said that it was better to have a wrong calendar than foreigners in China, in direct reference to the Jesuit missionaries.

Before briefly describing the events, it must be said that the Jesuit missionaries' efforts in the calendar were intended to demonstrate the superiority of Western astronomy, which they would use in their proselytizing work.

In 1660, Yang Guangxian began attacking Jesuit astronomy. The 1661 calendar had two major solar terms in one solar month, which was impossible in the old system that used the mean motion of the Sun but possible if the solar terms were determined by the true motion of the Sun. Yang argued in his denunciation that the traditional Chinese method should be followed to determine the solar terms.

After his first complaint was rejected, Yang submitted another memorial to the Emperor at the end of 1660, in which his attacks were more diversified. His complaint concerned the appearance of the word «West» in the name of Schall's work. Yang understood that this represented submission to the West and that the Jesuits intended to use the calendar for proselytizing.

Schall knew that the word *Xi* (West) was very unpopular, but with the new Qing dynasty, it was legitimately used, although the emperor, recognizing the symbolism of the word, used *Xin* (new) instead. Schall was aware that the words were crucial in Confucian cultural politics and used their symbolism to demonstrate the superiority of the Jesuit calendar.

In his second complaint, Yang returned to the subject of the calendar for the year 1661. Since it was an intercalary year and there was also a double month, there were two empty months, i.e. without major solar terms, the months 7 and 12. Yang asked why month 7 was chosen instead of the 12th. The emperor also ignored this second attack by Yang against the Jesuits.

In 1664, Yang Guangxian again denounced the Jesuits, accusing them of sedition and omitting the astronomical issues. Yang argued that if all humankind came from Adam and Eve, as the Christians claimed, all Chinese, including the emperor himself, originated in the West. This argument had consequences for the Manchu emperors of the Qing dynasty so the Jesuits were accused of sedition.

Yang's third complaint was accepted, and the Jesuits and their Chinese followers at the Astronomical Bureau were tried and acquitted of sedition but found guilty of their calendar and of an error in choosing the date of a prince's funeral.

Despite the court's ruling, the kingdom's regents convicted the Jesuits of sedition in April 1665 and pardoned their «errors» in the calendar. The day after the verdict was announced, a strong earthquake occurred in northern China, which was interpreted as a warning from Heaven that the verdict had been unjust. In response, the authorities released the Jesuits except for Schall, who was placed under

house arrest and died shortly afterwards. However, the Chinese officials of the Astronomical Bureau who had converted to Christianity were executed. After the trial of the Jesuits, Yang Guangxian was appointed director of the Astronomical Bureau despite not being an expert in astronomy and initially resisting taking up the post.

The new Kangxi Emperor ascended the throne in 1667 and soon became interested in calendar matters, a problem that Yang was unable to solve. The emperor ordered Yang and the head of the Muslim section of the Astronomical Bureau to cooperate with the Jesuits to make a good calendar. At the same time, the emperor asked for a prediction of the position of the Sun for the next day. The Belgian Jesuit Ferdinand Verbiest made a successful prediction, which meant the return of the Jesuits to the calendar office.

Yang, who was in a difficult position, argued that the old calendar, intended to give the dates of the auspices, should be followed. Verbiest replied that the important thing was the calendar's accuracy. In 1669, the Emperor requested a second and more complete astronomical test than the previous one, and again, Verbiest's predictions were successful.

As a result of the competition initiated by the Emperor, Yang was removed from his position as royal astronomer. This position was again filled by a Jesuit in the person of Verbiest, a position the Jesuits held for over a century. The calendar of 1669 and the following years were computed with the new techniques. Since then, the *dingqi* system has remained the method of calculating solar terms to this day.

Development of the *Shixian* calendar

The *Shixian* calendar introduced at the beginning of the Qing Dynasty was based on the *Xiyang Xinfa Lishu* prepared by Adam Schall, an unclear document. So, the emperor ordered the book to be rewritten in 1714. The work was completed in 1722, receiving the name *Lixiang Kaocheng* (Thorough Investigation Calendrical Asstronomy).

The *Lixiang Kaocheng* contains the theory and tables for calculating the motions of the Sun, Moon, and planets and information on eclipses. From 1726 onwards, the Imperial Astronomical Bureau calculated the calendar by the *Lixiang Kaocheng*, although the calendar prepared annually continued to be called the *Shixian.*

A solar eclipse in 1730 revealed errors in the *Lixiang Kaocheng,* necessitating its revision, which was completed in 1742 and renamed *Lixiang Kaocheng Houbian* (Supplements to the Thorough Investigation of Calendrical Astronomy).

Until 1914, the annual *Shixian Shu* calendar was calculated based on the *Lixiang Kaocheng Houbian*. In addition, the Astronomical Bureau occasionally published the work entitled *Wannian Shu* or *Wannian Li* (literally translated as Ten Thousand Year Calendar). The first *Wannian* was published in 1741 and contained information from 1624 to 1835. In 1787, the second *Wannian* was published during the reign of Emperor Qian Long; after his death, a new *Wannian* was published in 1799, which extended for the two hundred years of the new emperor Jia Qing's reign. Subsequently, after the accession of each new emperor, a new *Wannian* was published; this occurred in 1824, 1851, 1862, and 1876, the last being in 1910 with the last emperor Xuan Tong, with information up to 2108.

The data contained in the *Wannian* were the year of the sexagenary cycle, the

Year	Event
841 BCE	Reliable news about calendar dates.
589 BCE	Use of the Meton cycle.
104 BCE	This was the first major calendar reform. The no *zhongqi* rule was introduced, and the winter solstice must be in the 11th month.
619	Use of the true motion of the Moon.
1280	The *Shoushi Lishu* is adopted to prepare the calendar.
1383	The *Datong Lishu* is adopted to prepare the calendar.
1582	Mateo Ricci, the first Jesuit missionary, arrives in China.
1629	A committee is formed to improve the calendar.
1630	The Jesuit Adam Schall is appointed royal astronomer.
1635	The writing of the *Chongzhen Lishu* is completed.
1644	Schall prepared the work *Xiyang Xinfa Lishu,* which was used to prepare the calendar.
1645	Use of the true motion of the Sun.
1660	Yang Guangxian's first memorial against the Jesuits.
1661	Yang Guangxian's second memorial denouncing the Jesuits.
1664	Third and last memorial against the Jesuits.
1665	The Jesuits are condemned for their «errors» in the calendar.
1667	Accession to the throne of Emperor Kangxi who restores the Jesuits to the office of the calendar.
1669	The calendar is again prepared with the true Sun.
1722	Writing of the *Lixiang Kaocheng*.
1741	Publication of the first *Wannian Shu*.
1742	Writing of the *Lixiang Kaocheng Houbian*.
1787	Publication of the second *Wannian Shu* in the reign of Emperor Qian Long.
1799	Emperor Jia Qing's *Wannian Shu*.
1824	Emperor Dao Guang's *Wannian Shu*.
1851	Emperor Xian Feng's *Wannian Shu*.
1862	Emperor Tong Zhi's *Wannian Shu*.
1876	Emperor Guang Xu's *Wannian Shu*.
1910	Publication of the last *Wannian Shu* at the beginning of the reign of Emperor Xuan Tong (Puyi).
1912	The Gregorian calendar is declared official in China.
1912	The Central Observatory begins to publish the Calendar of the Republic of China annually.
1914	The calendar begins to be calculated using Western astronomical theory.
1924	Use of the mean time of the Beijing meridian.
1928	Creation of the Nanjing Institute of Astronomy with the responsibility of annually preparing the calendar.
1929	The government of the Kuomintang once again declares the Gregorian calendar official.
1950	Creation of the Purple Mountain Observatory with the responsibility of preparing the calendar.
1953	The government declares the calendar published by the Purple Mountain Observatory official.
1978	Unification of the calendar throughout China.

1984	The Newcomb Sun tables are no longer used to calculate the calendar.
1989	Taiwan accepts the calendar prepared by the Purple Mountain Observatory as its official calendar.
2013	The *Wannian Shu* is no longer used in the Chinese diaspora.
2017	The government publishes the rules for preparing the traditional calendar.

Table 1.15.- **MAJOR EVENTS IN THE HISTORY OF THE CHINESE CALENDAR.**

months of the year with an indication of long, short, or intercalary (i.e., the lunisolar calendar), the day of the sexagenary cycle of the 1st, 11th, and 21st of each month, and the date and time of each solar term.

The *Wannian Shu* was calculated using a simplified method, while the *Shixian Shu* was calculated using the complete theory given in the *Houbian*. For this reason, there are discrepancies between the two calendars, especially regarding the sequence of long and short months. In addition, the *Houbian* was not sufficiently accurate. It should be noted that when comparing dates calculated by current methods with the calendars of the Qing Dynasty, the *Houbian* used local time in Beijing.

Recent developments

Following the triumph of the 1911 revolution, the government adopted the Gregorian calendar from 1 January 1912, at least for official purposes. The first president of the Republic, Sun Yat-sen, sent telegrams to all regions announcing that the 13th day of the 11th month of the year 6409 of the reign of the Yellow Emperor Huangdi would correspond to 1 January 1912. The new Republic created the Central Observatory, thereby abolishing the Imperial Astronomical Bureau, which had been in charge of the calendar during the Qing dynasty.

The population continued to use the traditional calendar, so the status of the Gregorian calendar was unclear until 1921. After the reconstitution of the Chinese Republic led by the Kuomintang, the Gregorian calendar was adopted again from 1 January 1929.

From 1912, the Central Observatory published the «Calendar of the Republic of China» annually, essentially the Gregorian calendar but containing details of the lunisolar calendar. Until 1914, the calendar was still calculated by the *Lixiang Kaocheng Houbian*. However, from 1914 onwards, Western methods came into use, specifically the Newcomb Sun Tables, with an accuracy of one arcsecond, which was in use until 1984.

Years later, in 1928, the Nanjing Institute of Astronomy was established, which replaced the Central Observatory in preparing the calendar. In 1950, the Purple Mountain Observatory in Nanjing, near Shanghai, was put into operation and has had jurisdiction over the calendar since its creation.

The traditional lunisolar calendar remained in use despite the 1912 decision to officially use the Gregorian calendar. For this reason, its use was formally abolished in 1929, but lunar phases and solar terms continued to be calculated.

During the Qing Dynasty, the time used in the calendar was the local apparent time of Beijing. In 1924, the time changed to Beijing's mean time, and finally, in 1929,

the standard time zone where Beijing is located was used, Greenwich Mean Time plus 8 hours.

The last *Wannian* extended its information to 2108, but as we have said, it contained errors. Nevertheless, this *Wannian* was widely disseminated, and calendar printers continued to use it, which was confused when it diverged from the official calendar prepared by the Purple Mountain Observatory (PMO).

The *Wannian* of 1953 contained errors, so two calendars appeared in China, each giving different dates for the seventh lunar month. For the *Wannian*, it began on August 9, and for the calendar prepared by PMO, it began on August 10.

In 1978, two different versions of the calendar were published again. Hong Kong and southern China followed the *Wannian* calendar, which continued to use the apparent time of the Beijing meridian. As a result, the eighth month began on September 3, according to the PMO calculations and on September 2, according to the *Wannian*. This divergence affected the celebration of Mid Autumn, which falls on the 15th day of the eighth month. After this incident, the government reaffirmed the official status of the calendar prepared by the PMO, which has been followed in Hong Kong ever since.

A final discrepancy occurred in 1989 when a calendar based on the latest *Wannian* was published in Taiwan, which differed from the official Purple Mountain Observatory calendar. The director of the national observatory of Taiwan quickly announced through the press that the PMO had published the correct calendar.

Outside China, some publishers continued to use the *Wannian Shu* to prepare the calendar until 2013. From this date on, the Chinese calendar was definitively unified worldwide.

The latest episode in our brief history occurred in 2017 when the government published the document GB/T 33661-2017, which establishes how to calculate the Chinese lunisolar calendar, thus reconciling tradition with modern astronomical concepts.

Finally, it should be noted that the Chinese calendar spread to Korea, Japan, Hong Kong, Macao, Taiwan, and Vietnam. In Japan, the Gregorian calendar is used exclusively, but wall calendars still use the first day of the lunar month. In Malaysia, Laos, Singapore, and Mauritius, the Chinese New Year is a public holiday.

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Appendix I

Simplified Chinese calendar for the years 2025-2075

Simplified Chinese calendar for the years 2025-2075

Below is the simplified calendar for the years 2025 to 2075. The meaning of the columns is as follows:

The header contains the associated Gregorian year, indication of normal or intercalary year, sexagenary cycle of the year, cycle number and duration of the year.
* *Lunar month*. It numbers the lunar months from 1 to 12. The intercalary months receive the same numbering as the previous month with the addition of the letter «i».
* *Gregorian date of the lunar month*. Gregorian calendar date of the first day of the lunar month.
* *Duration*. Days that the lunar month lasts.
* *Day of the week*. The day of the week on which the lunar month begins. 1 is Sunday, and 7 is Saturday.
* *Gregorian date of the solar term*. Gregorian date on which the Sun is in the solar term. Each year has 12 solar terms, numbered from 1 to 12.

Lunar month	Gregorian date lunar month	Duration	Day week	Gregorian date solar term	Lunar month	Gregorian date lunar month	Duration	Day week	Gregorian date solar term	Lunar month	Gregorian date lunar month	Duration	Day week	Gregorian date solar term
2025/Intercalary/*Yi Si*/42/384					2026/Common/*Bing Wu*/43/354					2027/Common/*Ding Wei*/45/354				
1	2025/01/29	30	4	2025/02/18	1	2026/02/17	30	3	2026/02/18	1	2027/02/06	30	7	2027/02/19
2	2025/02/28	29	6	2025/03/20	2	2026/03/19	29	5	2026/03/20	2	2027/03/08	30	2	2027/03/21
3	2025/03/29	30	7	2025/04/20	3	2026/04/17	30	6	2026/04/20	3	2027/04/07	29	4	2027/04/20
4	2025/04/28	29	2	2025/05/21	4	2026/05/17	29	1	2026/05/21	4	2027/05/06	30	5	2027/05/21
5	2025/05/27	29	3	2025/06/21	5	2026/06/15	29	2	2026/06/21	5	2027/06/05	29	7	2027/06/21
6	2025/06/25	30	5	2025/07/22	6	2026/07/14	30	3	2026/07/23	6	2027/07/04	29	1	2027/07/23
6i	2025/07/25	29	6		7	2026/08/13	29	5	2026/08/23	7	2027/08/02	30	2	2027/08/23
7	2025/08/23	30	7	2025/08/23	8	2026/09/11	29	6	2026/09/23	8	2027/09/01	29	4	2027/09/23
8	2025/09/22	29	2	2025/09/23	9	2026/10/10	30	7	2026/10/23	9	2027/09/30	29	5	2027/10/23
9	2025/10/21	20	3	2025/10/23	10	2026/11/09	30	2	2026/11/22	10	2027/10/29	30	6	2027/11/22
10	2025/11/20	20	5	2025/11/22	11	2026/12/09	30	4	2026/12/22	11	2027/11/28	30	1	2027/12/22
11	2025/12/20	30	7	2025/12/21	12	2027/01/08	29	6	2027/01/20	12	2027/12/28	29	3	2027/01/20
12	2026/01/19	29	1	2026/01/20										
2028/Intercalary/*Wu Shen*/45/384					2029/Common/*Ji You*/46/355					2030/Common/*Geng Xu*/47/354				
1	2028/01/26	30	4	2028/02/19	1	2029/02/13	30	3	2029/02/18	1	2030/02/03	29	1	2030/02/18
2	2028/02/25	30	6	2028/03/20	2	2029/03/15	30	5	2029/03/20	2	2030/03/04	30	2	2030/03/20
3	2028/03/26	30	1	2028/04/19	3	2029/04/14	29	7	2029/04/20	3	2030/04/03	29	4	2030/04/20
4	2028/04/25	29	3	2028/05/20	4	2029/05/13	30	1	2029/05/21	4	2030/05/02	30	5	2030/05/21
5	2028/05/24	30	4	2028/06/21	5	2029/06/12	29	3	2029/06/21	5	2030/06/01	30	7	2030/06/21
5i	2028/06/23	29	6		6	2029/07/11	30	4	2029/07/22	6	2030/07/01	29	2	2030/07/23
6	2028/07/22	29	7	2028/07/22	7	2029/08/10	29	6	2029/08/23	7	2030/07/30	30	3	2030/08/23
7	2028/08/20	30	1	2028/08/22	8	2029/09/08	30	7	2029/09/23	8	2030/08/29	29	5	2030/09/23
8	2028/09/19	29	3	2028/09/22	9	2029/10/08	29	2	2029/10/23	9	2030/09/27	30	6	2030/10/23
9	2028/10/18	29	4	2028/10/23	10	2029/11/06	29	3	2029/11/22	10	2030/10/27	29	1	2030/11/22
10	2028/11/16	30	5	2028/11/22	11	2029/12/05	30	4	2029/12/21	11	2030/11/25	30	2	2030/12/22
11	2028/12/16	30	7	2028/12/21	12	2030/01/04	30	6	2030/01/20	12	2031/12/25	29	4	2031/01/20
12	2029/01/15	29	2	2029/01/20										

Lunar month	Gregorian date lunar month	Duration	Day week	Gregorian date solar term
2031/Intercalary/*Xin Hai*/48/384				
1	2031/01/23	29	5	2031/02/19
2	2031/02/21	30	6	2031/03/21
3	2031/03/23	30	1	2031/04/20
3i	2031/04/22	29	3	
4	2031/05/21	30	4	2031/05/21
5	2031/06/20	29	6	2031/06/21
6	2031/07/19	30	7	2031/07/23
7	2031/08/18	30	2	2031/08/23
8	2031/09/17	29	4	2031/09/23
9	2031/10/16	30	5	2031/10/23
10	2031/11/15	29	7	2031/11/22
11	2031/12/14	30	1	2031/12/22
12	2032/01/13	29	3	2032/01/20

Lunar month	Gregorian date lunar month	Duration	Day week	Gregorian date solar term
2032/Common/*Ren Xi*/49/355				
1	2032/02/11	30	4	2032/02/19
2	2032/03/12	29	6	2032/03/20
3	2032/04/10	29	7	2032/04/19
4	2032/05/09	30	1	2032/05/20
5	2032/06/08	29	3	2032/06/21
6	2032/07/07	30	4	2032/07/22
7	2032/08/06	30	6	2032/08/22
8	2032/09/05	29	1	2032/09/22
9	2032/10/04	30	2	2032/10/23
10	2032/11/03	30	4	2032/11/22
11	2032/12/03	29	6	2032/12/21
12	2033/01/01	30	7	2033/01/20

Lunar month	Gregorian date lunar month	Duration	Day week	Gregorian date solar term
2033/Intercalary/*Gui Chou*/50/384				
1	2033/01/31	29	2	2033/02/18
2	2033/03/01	30	3	2033/03/20
3	2033/03/31	29	5	2033/04/20
4	2033/04/29	29	6	2033/05/21
5	2033/05/28	30	7	2033/06/21
6	2033/06/27	29	2	2033/07/22
7	2033/07/26	30	3	2033/08/23
8	2033/08/25	29	5	2033/09/23
9	2033/09/23	30	6	2033/10/23
10	2033/10/23	30	1	2033/11/22
11	2033/11/22	30	3	2033/12/21
11i	2033/12/22	29	5	
12	2034/01/20	30	6	2034/01/20

Lunar month	Gregorian date lunar month	Duration	Day week	Gregorian date solar term
2034/Common/*Jia Yin*/51/354				
1	2034/02/19	29	1	2034/02/19
2	2034/03/20	30	2	2034/03/20
3	2034/04/19	29	4	2034/04/20
4	2034/05/18	29	5	2034/05/18
5	2034/06/16	30	6	2034/06/21
6	2034/07/16	29	1	2034/07/23
7	2034/08/14	30	2	2034/08/23
8	2034/09/13	29	4	2034/09/23
9	2034/10/12	30	5	2034/10/23
10	2034/11/11	30	7	2034/11/22
11	2034/12/11	29	2	2034/12/22
12	2035/01/09	30	3	2035/01/20

Lunar month	Gregorian date lunar month	Duration	Day week	Gregorian date solar term
2035/Common/*Yi Mao*/51/354				
1	2035/02/08	30	5	2035/02/19
2	2035/03/10	29	7	2035/03/21
3	2035/04/08	30	1	2935/04/20
4	2035/05/08	29	3	3025/05/21
5	2035/06/06	29	4	2035/06/21
6	2035/07/05	30	5	2035/07/23
7	2035/08/04	29	7	2035/08/23
8	2035/09/02	29	1	2035/09/23
9	2035/10/01	30	2	2035/10/23
10	2035/10/31	30	4	2035/11/22
11	2035/11/30	29	6	2035/12/22
12	2035/12/29	30	7	2036/01/20

Lunar month	Gregorian date lunar month	Duration	Day week	Gregorian date solar term
2036/Intercalary/*Bing Chen*/53/384				
1	2036/01/28	30	2	2036/02/19
2	2036/02/27	30	4	2036/03/20
3	2036/03/28	29	6	2036/04/19
4	2036/04/26	30	7	2036/05/20
5	2036/05/26	29	2	2036/06/21
6	2036/06/24	29	3	2036/07/22
6i	2036/07/23	30	4	
7	2036/08/22	29	6	2036/08/22
8	2036/09/20	29	7	2036/09/22
9	2036/10/19	30	1	2036/10/23
10	2036/11/18	29	3	2036/11/22
11	2036/12/17	30	4	2036/12/21
12	2037/01/16	30	6	2037/01/20

Lunar month	Gregorian date lunar month	Duration	Day week	Gregorian date solar term
2037/Common/*Ding Si*/54/354				
1	2037/02/15	30	1	2037/02/18
2	2037/03/17	30	3	2037/03/20
3	2037/04/16	29	5	2037/04/20
4	2037/05/15	30	6	2037/05/21
5	2037/06/14	29	1	2037/06/21
6	2037/07/13	29	2	2037/07/22
7	2037/08/11	30	3	2037/08/23
8	2037/09/10	29	5	2037/09/23
9	2037/10/09	29	6	2037/10/23
10	2037/11/07	30	7	2037/11/22
11	2037/12/07	29	2	2037/12/21
12	2038/01/05	30	3	2038/01/20

Lunar month	Gregorian date lunar month	Duration	Day week	Gregorian date solar term
2038/Common/*Wu Wu*/55/354				
1	2038/02/04	30	5	2038/02/18
2	2038/03/06	30	7	2038/03/20
3	2038/04/05	29	2	2038/04/20
4	2038/05/04	30	3	2038/05/21
5	2038/06/03	29	5	2038/06/21
6	2038/07/02	30	6	2038/07/23
7	2038/08/01	29	1	2038/08/23
8	2038/08/30	30	2	2038/09/23
9	2038/09/29	29	4	2038/10/23
10	2038/10/28	29	5	2038/11/22
11	2038/11/26	30	6	2038/12/22
12	2038/12/26	29	1	2039/01/20

Lunar month	Gregorian date lunar month	Duration	Day week	Gregorian date solar term
2039/Intercalary/*Ji Wei*/56/384				
1	2039/01/24	30	2	2039/02/19
2	2039/02/23	30	4	2039/03/25
3	2039/03/25	29	6	2039/04/23
4	2039/04/23	30	7	2039/05/21
5	2039/05/23	30	2	2039/06/21
5i	2039/06/22	29	4	
6	2039/07/21	30	5	2039/07/23
7	2039/08/20	29	7	2039/08/23
8	2039/09/18	30	1	2039/09/23
9	2039/10/18	29	3	2039/10/23
10	2039/11/16	30	4	2039/11/22
11	2039/12/16	29	6	2039/12/22
12	2040/01/14	29	7	2040/01/20

Lunar month	Gregorian date lunar month	Duration	Day week	Gregorian date solar term
2040/Common/*Geng Shen*/57/355				
1	2040/02/12	30	1	2040/02/19
2	2040/03/13	29	3	2040/03/20
3	2040/04/11	30	4	2040/04/19
4	2040/05/11	30	6	2040/05/20
5	2040/06/10	29	1	2040/06/21
6	2040/07/09	30	2	2040/07/22
7	2040/08/08	29	4	2040/08/22
8	2040/09/06	30	5	2040/09/22
9	2040/10/06	30	7	2040/10/23
10	2040/11/05	29	2	2040/11/22
11	2040/12/05	30	3	2040/12/21
12	2041/01/03	29	5	2041/01/20
2041/Common/*Xin You*/58/355				
1	2041/02/01	29	6	2041/02/18
2	2041/03/02	30	7	2041/03/20
3	2041/04/01	29	2	2041/04/20
4	2041/04/30	30	3	2041/05/20
5	2041/05/30	29	5	2041/06/21
6	2041/06/28	30	6	2041/07/22
7	2041/07/28	30	1	2041/08/23
8	2041/08/27	29	3	2041/09/22
9	2041/09/25	30	4	2041/10/23
10	2041/10/25	30	6	2041/11/22
11	2041/11/24	29	1	2041/12/21
12	2041/12/23	30	2	2042/01/20
2042/Intercalary/*Ren Xu*/59/384				
1	2042/01/22	29	4	2042/02/18
2	2042/02/20	30	5	2042/03/20
2i	2042/03/22	29	7	
3	2042/04/20	29	1	2042/04/20
4	2042/05/19	30	2	2042/05/21
5	2042/06/18	29	4	2042/06/21
6	2042/07/17	30	5	2042/07/23
7	2042/08/16	29	7	2042/08/23
8	2042/09/14	30	1	2042/09/23
9	2042/10/14	30	3	2042/10/23
10	2042/11/13	29	5	2042/11/22
11	2042/12/12	30	6	2042/12/22
12	2043/01/11	30	1	2043/01/20
2043/Common/Gui Hai/60/354				
1	2043/02/10	29	3	2043/02/19
2	2043/03/11	30	4	2043/03/21
3	2043/04/10	29	6	2043/04/20
4	2043/05/09	29	7	2043/05/21
5	2043/06/07	30	1	2043/06/21
6	2043/07/07	29	3	2043/07/23
7	2043/08/05	29	4	2043/08/23
8	2043/09/03	30	5	2043/09/23
9	2043/10/03	30	7	2043/10/23
10	2043/11/02	29	2	2043/11/22
11	2043/12/01	30	3	2043/12/22
12	2043/12/31	30	5	2044/01/20
2044/Intercalary/*Jia Zi*/1/384				
1	2044/01/30	30	7	2044/02/19
2	2044/02/29	29	2	2044/03/20
3	2044/03/29	30	3	2044/04/19
4	2044/04/28	29	5	2044/05/20
5	2044/05/27	29	6	2044/06/21
6	2044/06/25	30	7	2044/07/22
7	2044/07/25	29	2	2044/08/22
7i	2044/08/23	29	3	
8	2044/09/21	30	4	2044/09/22
9	2044/10/21	29	6	2044/10/23
10	2044/11/19	30	7	2044/11/22
11	2044/12/19	30	2	2044/12/21
12	2045/01/18	30	4	2045/01/20
2045/Common/*Yi Chou*/2/354				
1	2045/02/17	30	6	2045/02/18
2	2045/03/19	29	1	2045/03/20
3	2045/04/17	30	2	2045/04/19
4	2045/05/17	29	4	2045/05/20
5	2045/06/15	29	5	2045/06/21
6	2045/07/14	30	6	2045/07/22
7	2045/08/13	29	1	2045/08/23
8	2045/09/11	29	2	2045/09/22
9	2045/10/10	30	3	2045/10/23
10	2045/11/09	29	5	2045/11/22
11	2045/12/08	30	6	2045/12/21
12	2046/01/07	30	1	2046/01/20
2046/Common/*Bing Yin*/3/354				
1	2046/02/06	30	3	2046/02/18
2	2046/03/08	29	5	2046/03/20
3	2046/04/06	30	6	2046/04/20
4	2046/05/06	29	1	2046/05/21
5	2046/06/04	30	2	2046/06/21
6	2046/07/04	29	4	2046/07/22
7	2046/08/02	30	5	2046/08/23
8	2046/09/01	29	7	2046/09/23
9	2046/09/30	29	1	2046/10/23
10	2046/10/29	30	2	2046/11/22
11	2046/11/28	29	4	2046/12/22
12	2046/12/27	30	5	2047/01/20
2047/Intercalary/*Ding Mao*/4/384				
1	2047/01/26	30	7	2047/02/19
2	2047/02/25	29	2	2047/03/21
3	2047/03/26	30	3	2047/04/20
4	2047/04/25	30	5	2047/05/21
5	2047/05/23	29	7	2047/06/21
5i	2047/06/23	30	1	
6	2047/07/23	29	3	2047/07/23
7	2047/08/21	30	4	2047/08/23
8	2047/09/20	29	6	2047/09/23
9	2047/10/19	29	7	2047/10/23
10	2047/11/17	30	1	2047/11/22
11	2047/12/17	29	3	2047/12/22
12	2048/01/15	30	4	2048/01/20
2048/Normal/*Wu Chen*/5/354				
1	2048/02/14	29	6	2048/02/19
2	2048/03/14	30	7	2048/03/20
3	2048/04/13	30	2	2048/04/19
4	2048/05/13	29	4	2048/05/20
5	2048/06/11	30	5	2048/06/20
6	2048/07/11	30	7	2048/07/22
7	2048/08/10	29	2	2048/08/22
8	2048/09/08	30	3	2048/09/22
9	2048/10/08	29	5	2048/10/23
10	2048/11/06	29	6	2048/11/21
11	2048/12/05	30	7	2048/12/21
12	2049/01/04	29	2	2048/01/19

Lunar month	Gregorian date lunar month	Duration	Day week	Gregorian date solar term	Lunar month	Gregorian date lunar month	Duration	Day week	Gregorian date solar term	Lunar month	Gregorian date lunar month	Duration	Day week	Gregorian date solar term
2049/Common/*Ji Si*/6/355					2050/Intercalary/*Geng Wu*/7/384					2051/Common/*Xin Wei*/8/355				
1	2049/02/02	30	3	2049/02/18	1	2050/01/23	29	1	2050/02/18	1	2051/02/11	30	7	2051/02/19
2	2049/03/04	29	5	2049/03/20	2	2050/02/21	30	2	2050/03/20	2	2051/03/13	29	2	2051/03/20
3	2049/04/02	30	6	2049/04/19	3	2050/03/23	29	4	2050/04/20	3	2051/04/11	29	3	2051/04/20
4	2049/05/02	29	1	2049/05/20	3i	2050/04/21	30	5		4	2051/05/10	30	4	2051/05/21
5	2049/05/31	30	2	2049/06/21	4	2050/05/21	29	7	2050/05/21	5	2051/06/09	29	6	2051/06/21
6	2049/06/30	30	4	2049/07/22	5	2050/06/19	30	1	2050/06/21	6	2051/07/08	29	7	2051/07/23
7	2049/07/30	29	6	2049/08/22	6	2050/07/19	29	3	2050/07/22	7	2051/08/06	30	1	2051/08/23
8	2049/08/28	30	7	2049/09/22	7	2050/08/17	30	4	2050/08/23	8	2051/09/05	30	3	2051/09/23
9	2049/09/27	30	2	2049/10/23	8	2050/09/16	30	6	2050/09/23	9	2051/10/05	29	5	2051/10/23
10	2049/10/27	29	4	2049/11/22	9	2050/10/16	29	1	2050/10/23	10	2051/11/03	30	6	2051/11/22
11	2049/11/25	30	5	2049/12/21	10	2050/11/14	30	2	2050/11/22	11	2051/12/03	30	1	2051/12/22
12	2049/12/25	29	7	2050/01/20	11	2050/12/14	30	4	2050/12/22	12	2052/01/02	30	3	2052/01/20
					12	2051/01/13	29	6	2051/01/20					
2052/Intercalary/*Ren Shen*/9/384					2053/Common/*Gui You*/10/354					2054/Common/*Jia Xu*/11/354				
1	2052/02/01	29	5	2052/02/19	1	2053/02/19	29	4	2053/02/19	1	2054/02/08	29	1	2054/02/18
2	2052/03/01	30	6	2052/03/20	2	2053/03/20	30	5	2053/03/20	2	2054/03/09	30	2	2054/03/20
3	2052/03/31	29	1	2052/04/19	3	2053/04/19	29	7	2053/04/19	3	2054/04/08	30	4	2054/04/20
4	2052/04/29	29	2	2052/05/20	4	2053/05/18	29	1	2053/05/20	4	2054/05/08	29	6	2054/05/21
5	2052/05/28	30	3	2052/06/20	5	2053/06/16	30	2	2053/06/21	5	2054/06/06	29	7	2054/06/21
6	2052/06/27	29	5	2052/07/22	6	2053/07/22	29	4	2053/07/22	6	2054/07/05	30	1	2054/07/22
7	2052/07/26	29	6	2052/08/22	7	2053/08/14	29	5	2053/08/22	7	2054/08/04	29	3	2054/08/23
8	2052/08/24	30	7	2052/09/22	8	2053/09/22	30	6	2053/09/22	8	2054/09/02	29	4	2054/09/23
8i	2052/09/23	29	2		9	2053/10/23	29	1	2053/10/23	9	2054/10/01	30	5	2054/10/23
9	2052/10/22	30	3	2052/10/23	10	2053/11/22	30	2	2053/11/22	10	2054/10/31	29	7	2054/11/22
10	2052/11/21	30	5	2052/11/21	11	2053/12/21	30	4	2053/12/21	11	2054/11/29	30	1	2054/12/22
11	2052/12/21	30	7	2052/12/21	12	2054/01/20	30	6	2054/01/20	12	2054/12/29	30	3	2054/01/20
12	2053/01/20	30	2	2053/01/19										
2055/Intercalary/*Yi Hai*/12/383					2056/Common/*Bing Zi*/13/355					2057/Common /*Ding Chou*/14/354				
1	2055/01/28	29	5	2055/02/19	1	2056/02/15	30	3	2056/02/19	1	2057/02/04	29	1	2057/02/18
2	2055/02/26	30	6	2055/03/20	2	2056/03/16	30	5	2056/03/20	2	2057/03/05	30	2	2057/03/20
3	2055/03/28	30	1	2055/04/20	3	2056/04/15	30	7	2056/04/19	3	2057/04/04	30	4	2057/04/19
4	2055/04/27	29	3	2055/05/21	4	2056/05/15	29	2	2056/05/20	4	2057/05/04	29	6	2057/05/20
5	2055/05/26	30	4	2055/06/21	5	2056/06/13	30	3	2056/06/20	5	2057/06/02	30	7	2057/06/21
6	2055/06/25	29	6	2055/07/23	6	2056/07/13	29	5	2056/07/22	6	2057/07/02	29	2	2057/07/22
6i	2055/07/24	30	7		7	2056/08/11	30	6	2056/08/22	7	2057/07/31	30	3	2057/08/22
7	2055/08/23	29	2	2055/08/23	8	2056/09/10	29	1	2056/09/22	8	2057/08/39	30	5	2057/09/22
8	2055/09/21	29	3	2055/09/23	9	2056/10/09	29	2	2056/10/23	9	2057/09/29	29	7	2057/10/23
9	2055/10/20	30	4	2055/10/23	10	2056/11/07	30	3	2056/11/21	10	2057/10/28	29	1	2057/11/22
10	2055/11/19	29	6	2055/11/22	11	2056/12/07	29	5	2056/12/21	11	2057/11/26	30	2	2057/12/21
11	2055/12/18	30	7	2055/12/22	12	2057/01/05	30	6	2057/01/19	12	2057/12/26	20	4	2058/01/20
12	2056/01/17	29	2	2056/01/20										

Lunar month	Gregorian date lunar month	Duration	Day week	Gregorian date solar term
2058/Intercalary/*Wu Yin*/15/384				
1	2058/01/24	30	5	2058/02/18
2	2058/02/23	29	7	2058/03/20
3	2058/03/24	30	1	2058/04/20
4	2058/04/23	29	3	2058/05/21
4i	2058/05/22	30	4	
5	2058/06/21	29	6	2058/06/21
6	2058/07/20	30	7	2058/07/22
7	2058/08/19	30	2	2058/08/23
8	2058/09/18	29	4	2058/09/23
9	2058/10/17	30	5	2058/10/23
10	2058/11/16	30	7	2058/11/22
11	2058/12/16	29	2	2058/12/21
12	2059/01/14	29	3	2059/01/20
2059/Common/*Ji Mao*/16/355				
1	2059/02/12	30	4	2059/02/19
2	2059/03/14	29	6	2059/03/20
3	2059/04/12	30	7	2059/04/20
4	2059/05/12	29	2	2059/05/21
5	2059/06/10	30	3	2059/06/21
6	2059/07/10	29	5	2059/07/23
7	2059/08/08	30	6	2059/08/23
8	2059/09/07	29	1	2059/09/23
9	2059/10/06	30	2	2059/10/23
10	2059/11/05	30	4	2059/11/22
11	2059/12/05	30	6	2059/12/22
12	2060/01/04	29	1	2060/01/20
2060/Common/*Geng Chen*/17/354				
1	2060/02/02	30	2	2060/02/19
2	2060/03/03	29	4	2060/03/20
3	2060/04/01	29	5	2060/04/19
4	2060/04/20	30	6	2060/05/20
5	2060/05/30	29	1	2060/06/20
6	2060/06/28	29	2	2060/07/22
7	2060/07/27	30	3	2060/08/22
8	2060/08/26	29	5	2060/09/22
9	2060/09/24	30	6	2060/10/22
10	2060/10/24	30	1	2060/11/21
11	2060/11/23	30	3	2060/12/21
12	2060/12/23	29	5	2061/01/19
2061/Intercalary/*Xin Si*/18/384				
1	2061/01/21	30	6	2061/02/18
2	2061/02/20	30	1	2061/03/20
3	2061/03/22	29	3	2061/04/19
3i	2061/04/20	29	4	
4	2061/05/19	30	5	2061/05/20
5	2061/06/18	29	7	2061/06/21
6	2061/07/17	29	1	2061/07/22
7	2061/08/15	30	2	2061/08/22
8	2061/09/14	29	4	2061/09/22
9	2061/10/13	30	5	2061/10/23
10	2061/11/12	30	7	2061/11/22
11	2061/12/12	30	2	2061/12/21
12	2062/01/11	29	4	2062/01/20
2062/Common/*Ren Wu*/19/354				
1	2062/02/09	30	5	2062/02/18
2	2062/03/11	30	7	2062/03/20
3	2062/04/10	29	2	2062/04/20
4	2062/05/09	29	3	2062/05/21
5	2062/06/07	30	4	2062/06/21
6	2062/07/07	29	6	2062/07/22
7	2062/08/05	29	7	2062/08/23
8	2062/09/03	30	1	2062/09/23
9	2062/10/02	29	3	2062/10/23
10	2062/11/01	30	4	2062/11/22
11	2062/12/01	30	6	2062/12/21
12	2062/12/31	28	1	2063/01/20
2063/Intercalary/*Gui Wei*/20/384				
1	2063/01/29	30	2	2063/02/18
2	2063/02/28	30	4	2063/03/20
3	2063/03/30	29	6	2063/04/20
4	2063/04/28	30	7	2063/05/21
5	2063/05/28	29	2	2063/06/21
6	2063/06/26	30	3	2063/07/23
7	2063/07/26	29	5	2063/08/23
7i	2063/08/24	29	6	
8	2063/09/22	30	7	2063/09/23
9	2063/10/22	29	2	2063/10/23
10	2063/11/20	30	3	2063/11/22
11	2063/12/20	29	5	2063/12/22
12	2064/01/18	30	8	2064/01/20
2064/Common/*Jia Shen*/21/354				
1	2064/02/17	30	1	2064/02/19
2	2064/03/18	30	3	2064/03/20
3	2064/04/17	29	5	2064/04/19
4	2964/05/16	30	6	2064/05/20
5	2064/06/15	29	1	2064/06/20
6	2064/07/14	30	2	2064/07/22
7	2064/08/13	29	4	2064/08/22
8	2064/09/11	29	5	2064/09/22
9	2064/10/10	30	6	2064/10/22
10	2064/11/09	29	1	2064/11/21
11	2064/12/08	30	2	2064/12/21
12	2065/01/07	29	4	2065/01/19
2065/Common/*Yi You*/22/355				
1	2065/02/05	30	5	2065/02/18
2	2065/03/07	30	7	2065/03/20
3	2065/04/06	29	2	2065/04/19
4	2065/05/05	30	3	2065/05/20
5	2065/06/04	30	5	2065/06/21
6	2065/07/04	29	7	2065/07/22
7	2065/08/02	30	1	2065/08/22
8	2065/09/01	29	3	2065/09/22
9	2065/09/29	29	4	2065/10/23
10	2065/10/29	30	5	2065/11/22
11	2065/11/28	29	7	2065/12/21
12	2065/12/27	30	1	2066/01/20
2066/Intercalary /*Bing Xu*/23/384				
1	2066/01/26	29	3	2066/02/18
2	2066/02/24	30	4	2066/03/20
3	2066/03/26	29	6	2066/04/20
4	2066/04/24	30	7	2066/05/21
5	2066/05/24	30	2	2066/06/21
5i	2066/06/23	29	4	
6	2066/07/22	30	5	2066/07/22
7	2066/08/21	29	7	2066/08/23
8	2066/09/19	30	1	2066/09/23
9	2066/10/19	29	3	2066/10/23
10	2066/11/17	30	4	2066/11/22
11	2066/12/17	29	6	2066/12/21
12	2067/01/15	30	7	2067/01/20

Lunar month	Gregorian date lunar month	Duration	Day week	Gregorian date solar term	Lunar month	Gregorian date lunar month	Duration	Day week	Gregorian date solar term	Lunar month	Gregorian date lunar month	Duration	Day week	Gregorian date solar term
2067/Common/*Ding Hai*/24/354					2068/Common/*Wu Zi*/25/355					2069/Interalary/*Ji Chou*/26/384				
1	2067/02/14	29	2	2067/02/18	1	2068/02/03	30	6	2068/02/19	1	2069/01/23	29	4	2069/02/18
2	2067/03/15	30	3	2067/03/20	2	2068/03/04	29	1	2068/03/20	2	2069/02/21	30	5	2069/03/20
3	2067/04/14	29	5	2067/04/20	3	2068/04/02	30	2	2068/04/19	3	2069/03/23	29	7	2069/04/19
4	2067/05/13	30	6	2067/05/21	4	2068/05/02	29	4	2068/05/20	4	2069/04/21	30	1	2069/05/20
5	2067/06/12	29	1	2067/06/21	5	2068/05/31	29	5	2068/06/20	4i	2069/05/21	29	3	
6	2067/07/11	30	2	2067/07/23	6	2068/06/29	30	6	2068/07/22	5	2069/06/19	29	4	2069/06/21
7	2067/08/10	30	4	2067/08/23	7	2068/07/29	30	1	2068/08/22	6	2069/07/18	30	5	2069/07/22
8	2067/09/09	29	6	2067/09/23	8	2068/08/28	29	3	2068/09/22	7	2069/08/17	29	7	2069/08/22
9	2067/10/08	30	7	2067/10/23	9	2068/09/26	30	4	2068/10/22	8	2069/09/15	30	1	2069/09/22
10	2067/11/07	29	2	2067/11/22	10	2068/10/26	30	6	2068/11/21	9	2069/10/15	30	3	2069/10/23
11	2067/12/06	30	3	2067/12/22	11	2068/11/25	29	1	2068/12/21	10	2069/11/14	30	5	2069/11/22
12	2067/01/05	29	5	2068/01/20	12	2068/12/24	30	2	2069/01/19	11	2069/12/14	29	7	2069/12/21
										12	2070/01/12	30	1	2070/01/20
2070/Common/*Geng Yin*/27/354					2071/Intercalary/*Sin Mao*/28/384					2072/Common/*Ren Chen*/29/354				
1	2070/02/11	29	3	2070/02/18	1	2071/01/31	30	7	2071/02/18	1	2072/02/19	30	6	2072/02/19
2	2070/03/12	30	4	2070/03/20	2	2071/03/02	29	2	2071/03/20	2	2072/03/20	29	1	2072/03/20
3	2070/04/11	29	6	2070/04/20	3	2071/03/31	30	3	2071/04/20	3	2072/04/18	30	2	2072/04/19
4	2070/05/10	30	7	2070/05/20	4	2071/04/30	29	5	2071/05/21	4	2072/05/18	29	4	2072/05/20
5	2070/06/09	29	2	2070/06/21	5	2071/05/29	30	6	2071/06/21	5	2072/06/16	30	5	2072/06/20
6	2070/07/08	29	3	2070/07/22	6	2071/06/28	29	1	2071/07/23	6	2072/07/16	29	7	2072/07/22
7	2070/08/06	30	4	2070/08/23	7	2071/07/27	29	2	2071/08/23	7	2072/08/14	29	1	2072/08/22
8	2070/09/05	29	6	2070/09/22	8	2071/08/25	30	3	2071/09/23	8	2072/09/12	30	2	2072/09/22
9	2070/10/04	30	7	2070/10/23	8i	2071/09/24	29	5		9	2072/10/12	29	4	2072/10/22
10	2070/11/03	30	2	2070/11/22	9	2071/10/23	30	6	2071/10/23	10	2072/11/10	30	5	2072/11/21
11	2070/12/03	29	4	2070/12/21	10	2071/11/22	29	1	2071/11/22	11	2072/12/10	29	7	2072/12/21
12	2071/01/01	30	5	2071/01/20	11	2071/12/21	30	2	2071/12/22	12	2073/01/08	30	1	2073/01/19
					12	2072/01/20	30	4	2072/01/20					
2073/Common/*Gui Si*/30/354					2065/Intercalary/*Jia Wu*/31/384					2075/Common /*Yi Wei*/32/384				
1	2073/02/07	30	3	2073/02/18	1	2074/01/27	30	7	2074/02/18	1	2075/02/15	30	6	2075/02/18
2	2074/03/09	29	5	2073/03/09	2	2074/02/26	29	2	2074/03/20	2	2075/03/17	29	1	2075/03/20
3	2073/04/07	30	6	2073/04/19	3	2074/03/27	30	3	2074/04/20	3	2975/04/15	30	2	2075/04/20
4	2073/05/07	30	1	2073/05/20	4	2074/04/26	30	5	2074/05/20	4	2075/05/15	29	4	2075/05/21
5	2073/06/06	29	3	2073/06/21	5	2074/05/26	29	7	2074/06/21	5	2075/06/13	30	5	2075/06/21
6	2073/07/05	30	4	2073/07/22	6	2074/06/24	30	1	2074/07/22	6	2075/07/13	30	7	2075/07/22
7	2073/08/04	29	6	2073/08/22	6i	2074/07/24	29	3		7	2075/08/12	29	2	2075/08/23
8	2073/09/02	29	7	2073/09/22	7	2074/08/22	30	4	2074/08/23	8	2075/09/10	30	3	2075/09/23
9	2073/10/01	30	1	2073/10/23	8	2074/09/21	29	6	2074/09/22	9	2075/10/10	29	5	2075/10/23
10	2073/10/31	29	3	2073/11/22	9	2074/10/20	30	7	2074/10/23	10	2075/11/08	30	6	2075/11/22
11	2073/11/29	30	4	2073/12/21	10	2074/11/19	29	2	2074/11/22	11	2075/12/08	29	1	2075/12/22
12	2073/12/29	29	6	2074/01/20	11	2074/12/18	30	3	2074/12/21	12	2076/01/06	30	2	2076/01/20
					12	2075/01/17	29	5	2075/01/20					

Sexagenary cycle of days

The dates correspond to the beginning of each sexagenary cycle, from the year 2025 to 2076.

2025/02/24	2032/09/15	2040/02/06	2047/06/29	2054/11/19	2062/02/10	2069/07/03
2025/04/25	2032/11/14	2040/04/06	2047/08/28	2055/01/18	2062/04/11	2069/09/01
2025/06/24	2033/01/13	2040/06/05	2047/10/27		2062/06/10	2069/10/31
2025/08/23		2040/08/04	2047/12/26	2055/03/19	2062/08/09	2069/12/30
2025/10/22	2033/03/15	2040/10/03		2055/05/18	2062/10/08	
2025/12/21	2033/05/13	2040/12/02	2048/02/24	2055/07/17	2062/12/06	2070/02/28
	2033/07/12	2041/01/31	2048/04/24	2055/09/15		2070/04/29
2026/02/19	2033/09/10		2048/06/23	2055/11/14	2063/02/05	2070/06/28
2026/04/20	2033/11/09	2041/04/01	2048/08/22	2056/01/13	2063/04/06	2070/08/27
2026/06/19	2034/01/08	2041/05/31	2048/10/21		2063/06/05	2070/10/26
2026/08/18		2041/07/30	2048/12/20	2056/03/13	2063/08/04	2070/12/25
2026/10/17	2034/03/08	2041/09/28		2056/05/12	2063/10/03	
2026/12/16	2034/05/09	2041/11/27	2049/02/18	2056/07/11	2063/12/02	2071/02/23
	2034/07/07	2042/01/26	2049/04/19	2056/10/09		2071/04/24
2027/02/14	2034/09/05		2049/06/18	2056/11/08	2064/01/31	2071/06/23
2027/04/15	2034/11/04	2042/03/27	2049/08/17	2057/01/07	2064/03/31	2071/08/22
2027/06/14	2035/01/03	2042/05/26	2049/10/16		2064/05/30	2071/10/21
2027/08/13		2042/07/25	2049/12/15	2057/03/08	2064/07/29	2071/12/20
2027/10/12	2035/03/04	2042/09/23		2057/05/07	2064/09/27	
2027/12/11	2035/05/03	2042/11/22	2050/02/13	2057/07/06	2064/11/26	2072/02/18
	2035/07/02	2043/01/21	2050/04/14	2057/09/04	2065/01/25	2072/04/18
2028/02/09	2035/08/31		2050/06/13	2057/11/03		2072/06/17
2028/04/09	2035/10/30	2043/03/22	2050/08/12	2058/01/03	2065/03/26	2072/08/16
2028/06/08	2035/12/29	2043/05/21	2050/10/11		2065/05/25	2072/10/15
2028/08/07		2043/07/20	2050/12/10	2058/03/03	2065/07/24	2072/12/14
2028/10/06	2036/02/27	2043/09/18		2058/05/02	2065/09/22	
2028/12/05	2036/04/27	2043/11/17	2051/02/08	2058/07/01	2065/11/21	2073/02/12
	2036/06/26	2044/01/16	2051/04/09	2058/08/30	2066/01/20	2073/04/13
2029/02/04	2036/08/25		2051/06/08	2058/10/29		2073/06/12
2029/04/03	2036/10/24	2044/03/16	2051/08/07	2058/12/28	2066/03/21	2073/08/11
2029/06/02	2036/12/23	2044/05/15	2051/10/06		2066/05/20	2073/10/10
2029/08/01		2044/07/14	2051/12/05	2059/02/26	2066/07/19	2073/12/09
2029/11/30	2037/02/21	2044/09/12		2059/04/27	2066/09/17	
2030/01/29	2037/04/22	2044/11/11	2052/02/03	2059/06/26	2066/11/16	2074/02/07
	2037/06/21	2045/01/10	2052/04/03	2059/08/25	2067/01/13	2074/04/08
2030/03/30	2037/08/20		2052/06/02	2059/10/24		2074/06/07
2030/05/19	2037/10/19	2045/03/11	2052/08/01	2059/12/28	2067/03/16	2074/08/06
2030/07/28	2037/12/18	2045/05/10	2052/09/30		2067/05/15	2074/10/05
2030/09/26		2045/07/09	2052/11/29	2060/02/21	2067/07/14	2074/12/04
2030/11/25	2038/02/16	2045/09/07	2053/01/28	2060/04/21	2067/09/12	2075/02/02
2031/01/24	2038/04/17	2045/11/06		2060/06/20	2067/11/11	
	2038/06/16	2046/01/05	2053/03/29	2060/08/19	2068/01/10	2075/04/03
2031/03/26	2038/08/15		2053/05/28	2060/10/18		2075/06/02
2031/05/24	2038/10/14	2046/03/06	2053/07/27	2060/12/17	2068/03/10	2075/08/01
2031/07/23	2038/12/13	2046/05/05	2053/09/25		2068/05/09	2075/09/30
2031/09/21		2046/07/04	2053/11/24	2061/02/15	2068/07/08	2075/11/29
2031/11/20	2039/02/11	2046/09/02	2054/01/23	2061/04/16	2068/09/06	2076/01/28
2032/01/19	2039/04/12	2046/11/01		2061/06/15	2068/11/05	
	2039/06/11	2046/12/31	2054/03/24	2061/08/14	2069/01/04	2076/03/28
2032/03/19	2039/08/10		2054/05/23	2061/10/13		2076/05/27
2032/05/18	1039/10/09	2047/03/01	2054/07/22	2061/12/12	2069/03/05	2076/07/26
2032/07/15	2039/12/08	2047/04/30	2054/09/20		2069/05/04	2076/09/24

Sexagenary cycle of months

The sexagenary cycle of the first month of the year. The remaining months of the year follow the cycle in Table 2.9. Intercalary months do not belong to the sexagesimal cycle.

Year	Sexagenary cycle of the first month of the year
2025, 2030, 2035, 2040, 2045, 2050, 2055, 2060, 2065, 2070, 2075	*Wu Yin*
2026, 2031, 2036, 2041, 2046, 2051, 2056, 2061, 2066, 2071	*Geng Yin*
2027, 2032, 2037, 2042, 2047, 2052, 2057, 2062, 2067, 2072	*Ren Yin*
2028, 2033, 2038, 2043, 2048, 2053, 2058, 2063, 2068, 2073	*Jia Yin*
2029, 2034, 2039, 2044, 2049, 2054, 2059, 2064, 2069, 2074	*Bing Yin*

Appendix II

Astronomical tables of the Sun and the Moon

Coefficients L_0 for determining the longitude of the Sun
(VSOP87 theory simplified by Meeus, formulas 6.5 and 7.5)

Angles *B* and *C* are expressed in radians and *A* is in units of 10^{-8} radians (for longitude and latitude) and 10^{-8} astronomical units for distance.

	A	*B*	*C*
1	175 347 046	0	0
2	3 341 656	4.669 256 8	6 283.075 850 0
3	34 894	4.626 10	12 566.151 70
4	3 497	2.744 1	5 753.384 9
5	3 418	2.828 9	3.523 1
6	3 136	3.627 7	77 713.771 5
7	2 676	4.418 1	7 860.419 4
8	2 343	6.135 2	3 930.209 7
9	1 324	0.742 5	11 506.769 8
10	1 273	2.037 1	529.691 0
11	1 199	1.109 6	1 577.343 5
12	990	5.233	5 884.927
13	902	2.045	26.298
14	857	3.508	398.149
15	780	1.179	5 223.694
16	753	2.533	5 507.553
17	505	4.583	18 849.228
18	492	4.205	775.523
19	357	2.920	0.067
20	317	5.849	11 790 629
21	284	1.899	796.298
22	271	0.315	10 977.079
23	243	0.345	5 486.778
24	206	4.806	2 544.314
25	205	1.869	5 573.143
26	202	2.458	6 069.777
27	156	0.833	213.299
28	132	3.411	2 942.463
29	126	1.083	20.775
30	115	0.645	0.980

31	103	0.636	4 694.003
32	102	0.976	15 720.839
33	102	4.267	7.114
34	99	6.21	2 146.17
35	98	0.68	155.42
36	86	5.98	161 000.69
37	85	1.30	6 275.96
38	85	3.67	71 430.70
39	80	1.81	17 260.15
40	79	3.04	12 036.46
41	75	1.76	5 088.63
42	74	3.50	3 154.69
43	74	4.68	801.82
44	70	0.83	9 437.76
45	62	3.98	8 827.39
46	61	1.82	7 084.90
47	57	2.78	6 286.60
48	56	4.39	14 143.50
49	56	3.47	6 279.55
50	52	0.19	12 139.55
51	52	1.33	1 748.02
52	51	0.28	5 856.48
53	49	0.49	1 194.45
54	41	5.37	8 429.24
55	41	2.40	19 651.05
56	39	6.17	10 447.39
57	37	6.04	10 213.29
48	37	2.57	1 059.38
59	36	1.71	2 352.87
60	36	1.78	6 812.77
61	33	0.59	17 789.85
62	30	0.44	83 996.85
63	30	2.74	1 349.87
64	25	3.16	4 690.48

Coefficients L_1 for determining the longitude of the Sun

1	628 331 966 747	0	0
2	206 059	2.678 235	6 283.075 850
3	4 303	2.635 1	12 566.151 7
4	425	1.590	3.523
5	119	5.796	26.298
6	109	2.966	1 577.344
7	93	2.59	18 849.23
8	72	1.14	529.69
9	68	1.87	398.15
10	67	4.41	5 507.55
11	59	2.89	5 223.69
12	56	2.17	155.42
13	45	0.40	796.30
14	36	0.47	775.52
15	29	2.65	7.11
16	21	5.34	0.98
17	19	1.85	5 486.78
18	19	4.97	213.30
19	17	2.99	6 275.96
20	16	0.03	2 544,31
21	16	1.43	2 146.17
22	15	1.21	10 977.08
23	12	2.83	1 748.02
24	12	3.26	5 088.63
25	12	5.27	1 194.45
26	12	2.08	4 694.00
27	11	0.77	553.57
28	10	1.30	6 286.60
29	10	4.24	1 349.87
30	9	2.70	242.73
31	9	5.64	951.72
32	8	5,30	2 354.87
33	6	2.65	9 437.76
34	6	4.67	4.698

Coefficients L_2 for determining the longitude of the Sun

1	52 919	0	0
2	8 720	1.0721	6 283.075 8
3	309	0.867	12 566.152
4	27	0.05	3.52
5	16	5.19	26.30
6	16	3.68	155.42
7	10	0.76	18 849.23
8	9	2.06	77 713.77
9	7	0.83	775.52
10	5	4.66	1 577.34
11	4	1.03	7.11
12	4	3.44	5 573.14
13	3	5.14	796.30
14	3	6.05	5 507.55
15	3	1.19	242.73
16	3	6.12	529.69
17	3	0.31	398.15
18	3	2.28	553.57
19	2	4.38	5 223.69
20	2	3.75	0.98

Coefficients L_3 for determining the longitude of the Sun

1	289	5.844	06 283.076
2	35	0	0
3	17	5.49	12 566.15
4	3	5.20	155.42
5	1	4.72	3.52
6	1	5.30	18 849.23
7	1	5.97	242.73

Coefficients L_4 for determining the longitude of the Sun

1	114	3.142	0
2	8	4.13	6 283.08
3	1	3.84	12 566.15

Coefficients L_5 for determining the longitude of the Sun

1	1	3.14	0

Coefficients B_0 for determining the latitude of the Sun

1	280	3.199	84 334.662
2	102	5.422	5 507.553
3	80	3.88	5 223.69
4	44	3.70	2 352.87
5	32	4.00	1 577.34

Coefficients B_1 for determining the latitude of the Sun

1	9	3.90	5 507.55
2	6	1.73	5 223.69

Coefficients B_2 for determining the latitude of the Sun

1	100 013 989	0	0
2	1 670 700	3.098 463 5	6 283.075 850 0
3	13 956	3.055 25	12 566.151 70
4	3 084	5.198 5	77 713.771 5
5	1 628	1.173 9	5 753.384 9
6	1 576	2.846 9	7 860.419 4
7	925	5.453	11 506.770
8	542	4.564	3 930.210
9	472	3.661	5 884.927
10	346	0.964	5 507.553
11	329	5.900	5 223.694
12	307	0.299	5 573.143
13	243	4.273	11 790.629
14	212	5.847	1 577.344
15	186	5.022	10 977.079
16	175	3.012	18 849.228
17	110	5.055	5 486.778
18	98	0.89	6 069.78
19	86	5.69	15 720.84
20	86	1.27	161 000.69
21	65	0.27	17 260.15

22	63	0.92	529.69
23	57	2.01	83 996.85
24	56	5.24	71 430.70
25	49	3.25	2 544.31
26	47	2.58	775.52
27	45	5.54	9 437.76
28	43	6.01	6 275.96
29	39	5.36	4 694.00
30	38	2.39	8 827.39
31	37	0.83	19 651.05
32	37	4.90	12 139.55
33	36	1.67	12 036.46
34	35	1.84	2 942.46
35	33	0.24	7 084.90
36	32	0.18	5 088.63
37	32	1.78	398.15
38	28	1.21	6 286.60
39	28	1.90	6 279.55
40	26	4.59	10 447.39

Coefficients R_0 for determining the Earth-Sun distance

1	103 019	1.107 490	6 283.075 850
2	1 721	1.064 4	12 566.151 7
3	702	3.142	0
4	32	1.02	18 849.23
5	31	2.84	5.507.55
6	25	1.32	5 223.69
7	18	1.42	1 577.34
8	10	5.91	10 977.08
9	9	1.42	6 275.96
10	9	0.27	5 486.78

Coefficients R_1 for determining the Earth-Sun distance

1	4 359	5.784 6	6 283.075 8
2	124	5.579	12 566.152
3	12	3.14	0

Coefficients R_2 for determining the Earth-Sun distance

4	9	3.63	77 713.77
5	6	1.87	5.573.14
6	3	5.4	18 849.23

Coefficients R_3 for determining the Earth-Sun distance

1	52 145	4.273	6 283.076
2	7	3.92	12 566.15

Coefficients R_4 for determining the Earth-Sun distance

1	4	2.56	6 283.08

Coefficients for determining longitude, latitude and distance
from the Moon to the Earth
(Chapront's ELP-2000/82 theory simplified by Meeus, formula 10.5)

Periodic terms for the longitude L_i' and distance r_i' of the Moon
The unit is 0.000 001 degrees for L_i' and 0.001 km for r_i'

a_i	b_i	c_i	d_i	Coefficient of the sine	Coefficient of the cosine
j_i	k_i	l_i	m_i	L_i'	r_i'
0	0	1	0	6 288 774	-20 905 355
2	0	-1	0	1 274 027	-3 699 111
2	0	0	0	658 314	-2 955 968
0	0	2	0	213 618	-569 925
0	1	0	0	-185 116	48 888
0	0	0	2	-114 332	-3 149
2	0	-2	0	58 793	246 158
2	-1	-1	0	57 066	-152 138
2	0	1	0	53 322	-170 733
2	-1	0	0	45 758	-204 586
0	1	-1	0	-40 923	-129 620
1	0	0	0	-34 720	108 743
0	1	1	0	-30 383	104 755
2	0	0	-2	15 327	10 321
0	0	1	2	-12 528	
0	0	1	-2	10 980	79 661
4	0	-1	0	10 675	-34 782
0	0	3	0	10 034	-23 210
4	0	-2	0	8 548	-21 636
2	1	-1	0	-7 888	24 208
2	1	0	0	-6 766	30 824
1	0	-1	0	-5 163	-8 379
1	1	0	0	4 987	-16 675
2	-1	1	0	4 036	-12 831
2	0	2	0	3 994	-10 445
4	0	0	0	3 861	-11 650
2	0	-3	0	3 665	14 403
0	1	-2	0	-2 689	-7 003

a_i	b_i	c_i	d_i	Coefficient of the sine	Coefficient of the cosine
j_i	k_i	l_i	m_i	L_i'	r_i'
2	0	-1	2	-2 602	
2	-1	-2	0	2 390	10 056
1	0	1	0	-2 348	6 322
2	-2	0	0	2 236	-9 884
0	1	2	0	-2 120	5751
0	2	0	0	-2 069	
2	-2	-1	0	2 048	-4 950
2	0	1	-2	-1 773	4130
2	0	0	2	-1 595	
4	-1	-1	0	1 215	-3958
0	0	2	2	-1 110	
3	0	-1	0	-892	3 258
2	1	1	0	-810	2 616
4	-1	-2	0	759	-1 897
0	2	-1	0	-713	-2 117
2	2	-1	0	-700	2 354
2	1	-2	0	691	
2	-1	0	-2	596	
4	0	1	0	549	-1 423
0	0	4	0	537	-1 117
4	-1	0	0	520	-1 571
1	0	-2	0	-487	-1 739
2	1	0	-2	-399	
0	0	2	-2	-381	-4 421
1	1	1	0	351	
3	0	-2	0	-340	
4	0	-3	0	330	
2	-1	2	0	327	
0	2	1	0	-323	1 165
1	1	-1	0	299	
2	0	3	0	294	
2	0	-1	-2		8 752

Periodic Terms for the Moon's Latitude The unit is 0.000 001 degrees									
e_i	f_i	g_i	h_i	Coefficient of the sine λ_i'	e_i	f_i	g_i	h_i	Coefficient of the sine λ_i'
0	0	0	1	5 128 122	0	0	1	-3	777
0	0	1	1	280 602	4	0	-2	1	671
0	0	1	-1	277 693	2	0	0	-3	607
2	0	0	-1	173 237	2	0	2	-1	596
2	0	-1	1	55 413	2	-1	1	-1	491
2	0	-1	-1	46 271	2	0	-2	1	-451
2	0	0	1	32 573	0	0	3	-1	439
0	0	2	1	17 198	2	0	2	1	422
2	0	1	-1	9 266	2	0	-3	-1	421
0	0	2	-1	8 822	2	1	-1	1	-366
2	-1	0	-1	8 216	2	1	0	1	-351
2	0	-2	-1	4 324	4	0	0	1	331
2	0	1	1	4 200	2	-1	1	1	315
2	1	0	-1	-3 358	2	-2	0	-1	302
2	-1	-1	1	2 463	0	0	1	3	-283
2	-1	0	1	2 211	2	1	1	-1	-229
2	-1	-1	-1	2 065	1	1	0	-1	223
0	1	-1	-1	-1 870	1	1	0	1	223
4	0	-1	-1	1 828	0	1	-2	-1	-220
0	1	0	1	-1 794	2	1	-1	-1	-220
0	0	0	3	-1 749	1	0	1	1	-185
0	1	-1	1	-1 565	2	-1	-2	-1	181
1	0	0	1	-1 491	0	1	2	1	-177
0	1	1	1	-1 475	4	0	-2	-1	176
0	1	1	-1	-1 410	4	-1	-1	-1	166
0	1	0	-1	-1 344	1	0	1	-1	-164
1	0	0	-1	-1 335	4	0	1	-1	132
0	0	3	1	1 107	1	0	-1	-1	-119
4	0	0	-1	1 021	4	-1	0	-1	115
4	0	-1	1	833	2	-2	0	1	107

Appendix III

Julian Day

Julian day of a date in the Julian calendar

Let Y/M/D and H:M:S the date and time expressed in the Julian calendar; we call the auxiliary calendar date Y′/M′/D′, calculated by

$$\begin{aligned} D' &= D-1 \\ M' &= (M+9) \bmod 12 \\ Y' &= Y+4716-\text{int}\left(\frac{14-M}{12}\right), \end{aligned} \tag{A.1}$$

the associated Julian Day is

$$JD = \text{int}\left(\frac{1461Y'}{4}\right)+\text{int}\left(\frac{153M'+2}{5}\right)+D'-1401{,}5+\frac{1}{24}\left[H+\frac{1}{60}\left(M+\frac{S}{60}\right)\right].$$

Julian calendar date knowing the Julian Day

To determine the date and time of the Julian calendar knowing the Julian Day *JD*, we find the auxiliary magnitudes

$$\begin{aligned} T &= \text{int}\left\{\frac{\left\{4\left[\text{int}(JD+0.5)+1401\right]+3\right\} \bmod 1461}{4}\right\} \\ T' &= JD-0.5-\text{int}(JD-0.5), \end{aligned} \tag{A.2}$$

with which we determine the date in the auxiliary calendar

$$\begin{aligned} Y' &= \text{int}\left\{\frac{4\left[\text{int}(JD+0.5)+1401\right]+3}{1461}\right\} \\ M' &= \text{int}\left(\frac{5T+2}{153}\right) \\ D' &= \text{int}\left[\frac{(5T+2) \bmod 153}{5}\right]. \end{aligned} \tag{A.3}$$

Finally, the date in the Julian calendar and in time is

$$\begin{aligned} D &= D'+1 \\ M &= (M'+2) \bmod 12+1 \\ Y &= Y'-4716+\text{int}\left(\frac{14-M}{12}\right) \\ H &= \text{int}(24T') \\ M &= \text{int}\left[60(24T'-H)\right] \\ S &= 60\left\{60(24T'-H)-M\right\}. \end{aligned} \tag{A.4}$$

Julian Day of a date in the Gregorian calendar

Let Y/M/D and H:M:S the date and time expressed in the Gregorian calendar, by (A.1) we find the date of the auxiliary calendar and the corresponding Julian Day is

$$\begin{aligned} JD = \text{int}\left(\frac{1461Y'}{4}\right)+\text{int}\left(\frac{153M'+2}{5}\right)+D'-1401.5+ \\ +\frac{1}{24}\left[H+\frac{1}{60}\left(M+\frac{S}{60}\right)\right]-\text{int}\left\{\frac{3}{4}\left[1+\text{int}\left(\frac{Y'-6316}{100}\right)\right]\right\}-10. \end{aligned}$$

Gregorian calendar date knowing the Julian Day

Let *JD* the Julian day of a date in the Gregorian calendar, then the date in the Julian calendar that is equal to the searched date in the Gregorian calendar has the Julian Day

$$JD_j = JD + \text{int}\left\{\frac{3}{4}\left[1 + \text{int}\left(\frac{4JD - 9\ 222\ 033}{146\ 097}\right)\right]\right\} + 10,$$

known JD_j we find the date and time expressed in the Gregorian calendar by the formulas (A.2), (A.3) and (A.4).

Chronological Julian Day

We distinguish between astronomical and chronological Julian Days. The former defines a precise moment in time from which we can deduce the date and time of the day. The chronological Julian day is the astronomical Julian Day at 0 o'clock. Since the astronomical Julian Day begins at noon, the chronological Julian day is a half-integer number.

If JD_a is the astronomical Julian Day, the chronological Julian Day JD_c is

$$if\ JD_a - \text{int}(JD_a) < 0.5 \ \Rightarrow\ JD_c = \text{int}(JD_a) - 0.5$$
$$if\ JD_a - \text{int}(JD_a) \geq 0.5 \ \Rightarrow\ JD_c = \text{int}(JD_a) + 0.5$$

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